



Dynamics under location uncertainty: model derivation, modified transport and uncertainty quantification

Valentin Resseguier, Baylor Fox-Kemper, Etienne Mémin, Bertrand Chapron

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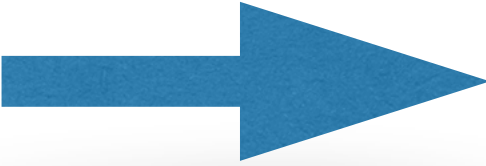
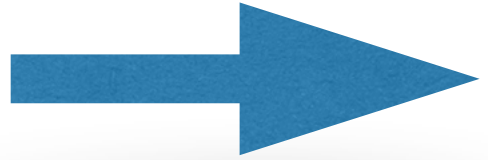
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Dynamics under location uncertainty: model derivation, modified transport and uncertainty quantification

Valentin Resseguier, Baylor Fox-Kemper
Etienne Memin, Bertrand Chapron



Motivations

- Rigorously identified subgrid dynamics effects
- Injecting likely small-scale dynamics
- Predict extreme events
- Quantification of modeling errors  Ensemble forecasts and data assimilation
- Studying different likely scenarios and attractors  Climate projections

Contents

I. Location uncertainty

II. SQG under moderate uncertainty

Part I

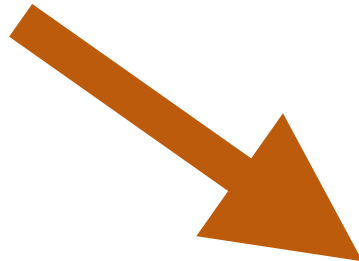
Location uncertainty

Adding random velocity

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

Adding random velocity

Resolved
large scales


$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

Adding random velocity

Resolved
large scales

$$\boldsymbol{v} = \boldsymbol{w} + \sigma \dot{\boldsymbol{B}}$$

White-in-time
small scales

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance
 tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Adding random velocity

Resolved
large scales

White-in-time
small scales

$$v = w + \sigma \dot{B}$$

References :

Mikulevicius and Rozovskii, 2004
 Flandoli, 2011

Memin, 2014
 Resseguier et al. 2017 a, b, c
 Chapron et al. 2017
 Cai et al. 2017

Holm, 2015
 Holm and Tyranowski, 2016
 Arnaudon et al., 2017

Cotter and al 2017
 Crisan et al., 2017
 Gay-Balmaz and Holm 2017

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\frac{D\Theta}{Dt} = 0$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

Large scales:

\boldsymbol{w}

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\sigma d\boldsymbol{B} (\sigma d\boldsymbol{B})^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \boldsymbol{w}^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \boxed{\text{Advection}} \quad w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta}_{\text{Drift correction}} + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Advection

Diffusion

Drift correction

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

The diagram illustrates the advection of a tracer Θ . It features a central equation with two colored boxes highlighting specific terms. The first box, labeled "Advection", is light blue and contains the term $w^* \cdot \nabla \Theta$, where w^* is circled in orange. The second box, labeled "Diffusion", is light green and contains the term $\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$. An orange arrow labeled "Drift correction" points to the w^* term. A purple arrow labeled "Multiplicative random forcing" points to the $\sigma \dot{B} \cdot \nabla \Theta$ term. The full equation is $\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$.

$$\partial_t \Theta + w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta = \nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)$$

Drift correction

Multiplicative random forcing

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{w^* \cdot \nabla \Theta + \sigma \dot{B} \cdot \nabla \Theta}_{\text{Advection}} = \underbrace{\nabla \cdot \left(\frac{1}{2} a \nabla \Theta \right)}_{\text{Diffusion}}$$

Drift correction

Multiplicative random forcing

Large scales:
 \boldsymbol{w}
 Small scales:
 $\sigma \dot{\boldsymbol{B}}$
 Variance
 tensor:
 $\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) =$
 $\frac{\mathbb{E}\{\sigma d\boldsymbol{B} (\sigma d\boldsymbol{B})^T\}}{dt}$

Advection of tracer Θ

$$\partial_t \Theta + \underbrace{\boldsymbol{w}^* \cdot \nabla \Theta}_{\text{Drift correction}} + \underbrace{\sigma \dot{\boldsymbol{B}} \cdot \nabla \Theta}_{\text{Multiplicative random forcing}} = \underbrace{\nabla \cdot \left(\frac{1}{2} \boldsymbol{a} \nabla \Theta \right)}_{\text{Diffusion}}$$

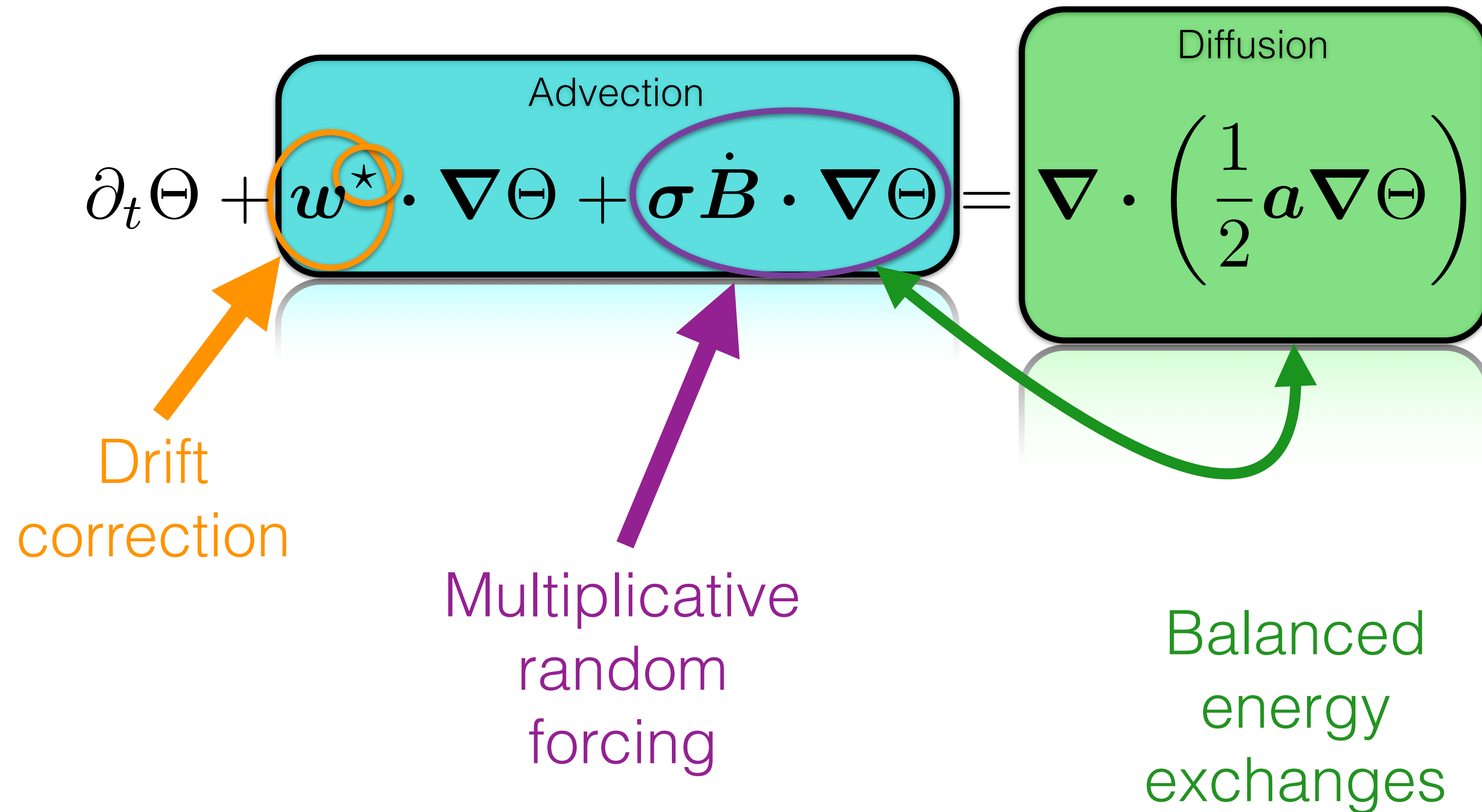
Drift correction

Multiplicative random forcing

Diffusion

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

Advection of tracer Θ



Large scales:

\boldsymbol{w}

Small scales:

$\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance
tensor:

$$\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$$

Derived random models

Conservations
(mass, linear
momentum, ...)

$$\frac{D}{Dt}$$

Navier-Stokes

Derived random models

Large scales:

w

Small scales:

$\sigma \dot{B}$

Variance
tensor:

$$a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$$

Conservations
(mass, linear
momentum, ...)

$$\frac{D}{Dt}$$

Navier-Stokes

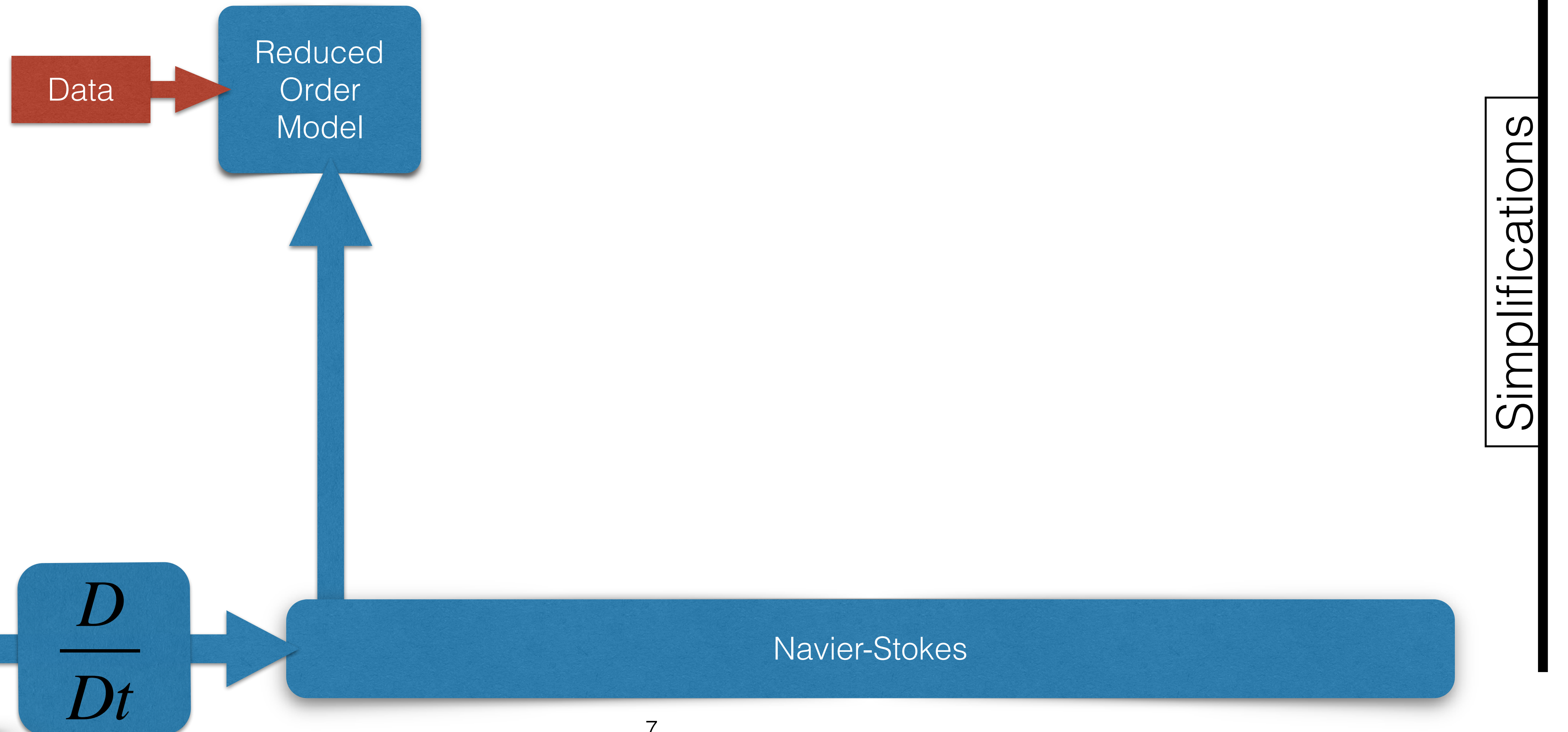
Simplifications

Derived random models

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

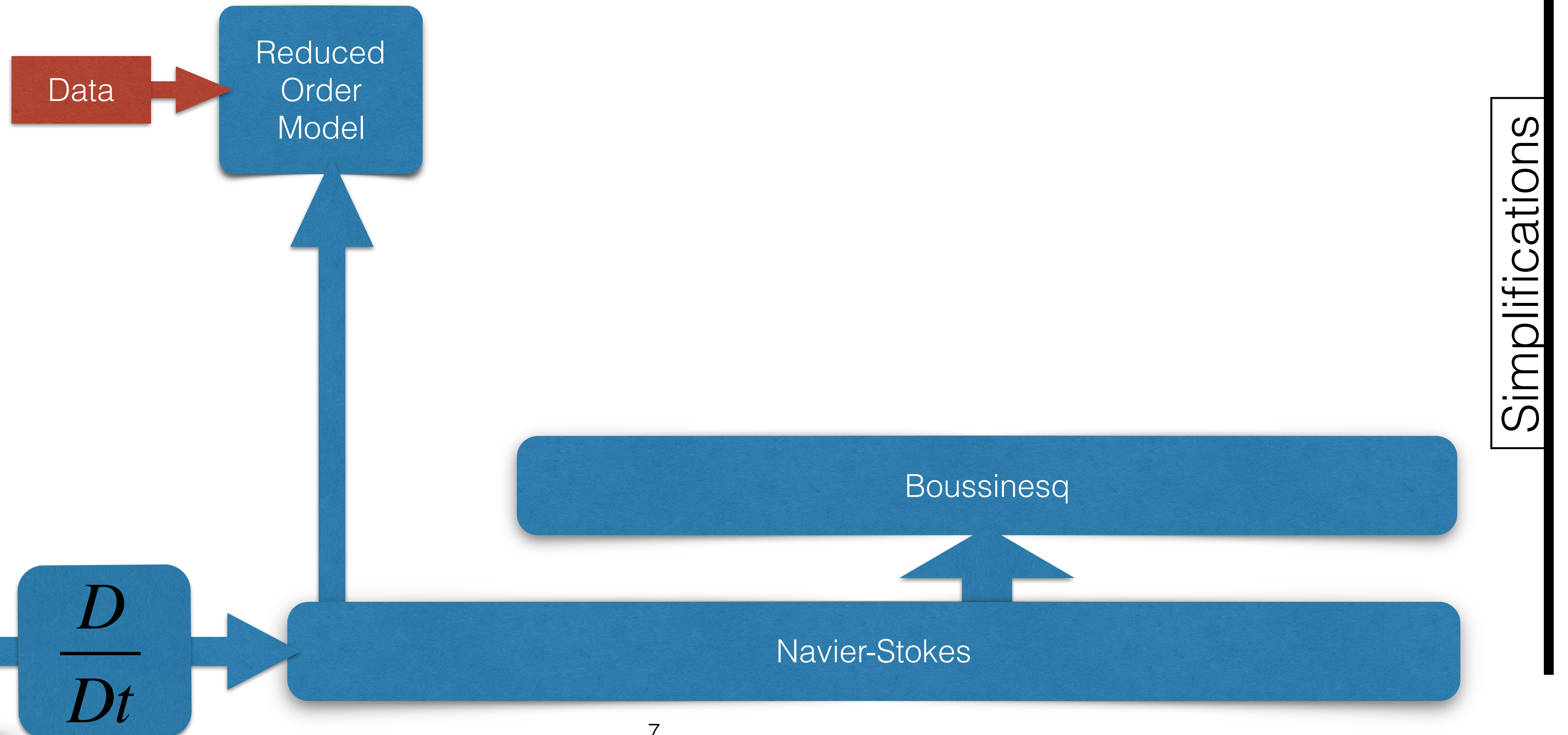


Derived random models

Large scales:
 \boldsymbol{w}

Small scales:
 $\boldsymbol{\sigma} \dot{\boldsymbol{B}}$

Variance
tensor:
 $\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) =$
 $\frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

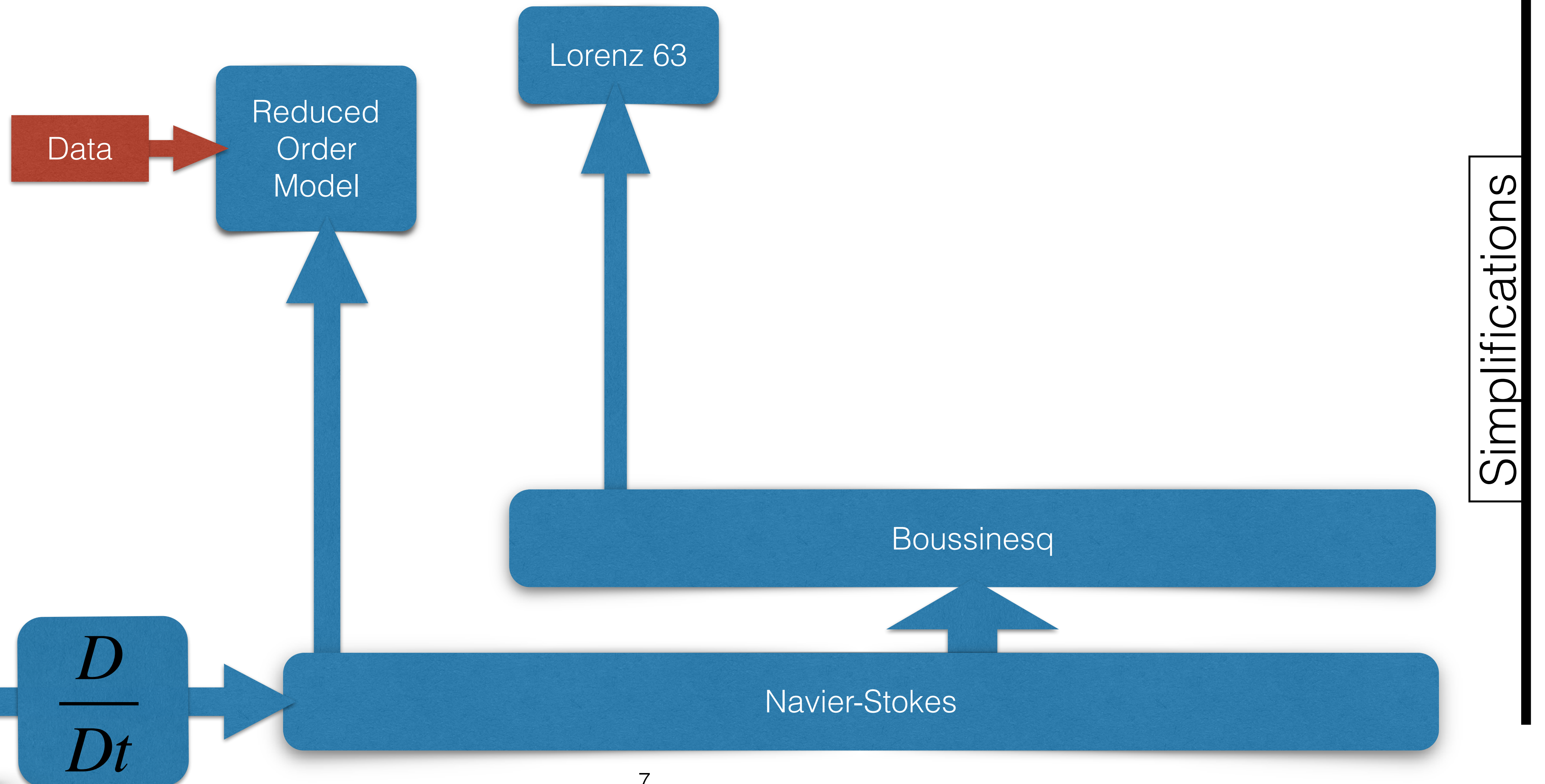


Derived random models

Large scales:
 w

Small scales:
 $\sigma \dot{B}$

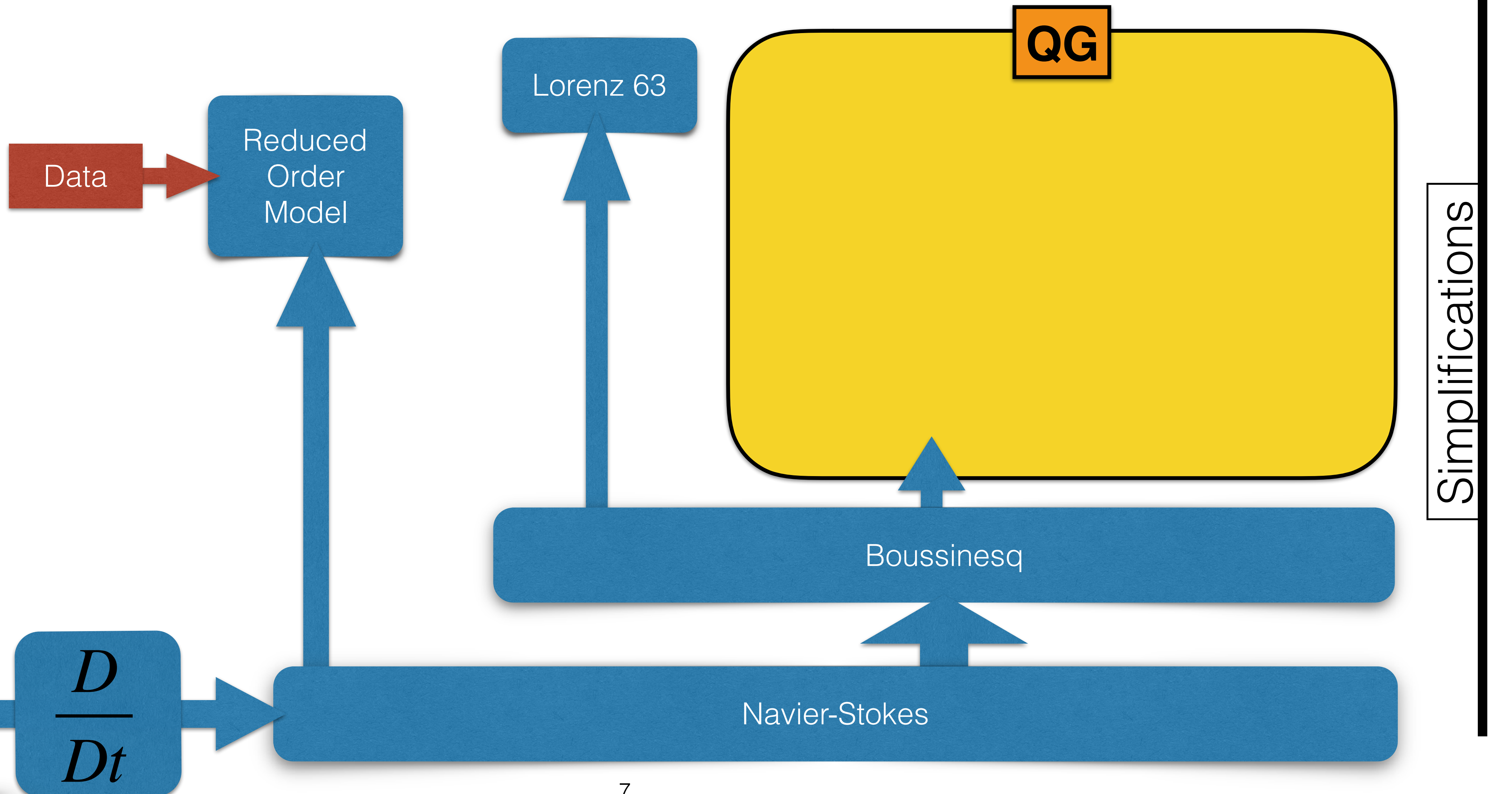
Variance
tensor:
 $a = a(x, x) =$
 $\frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



Derived random models

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$

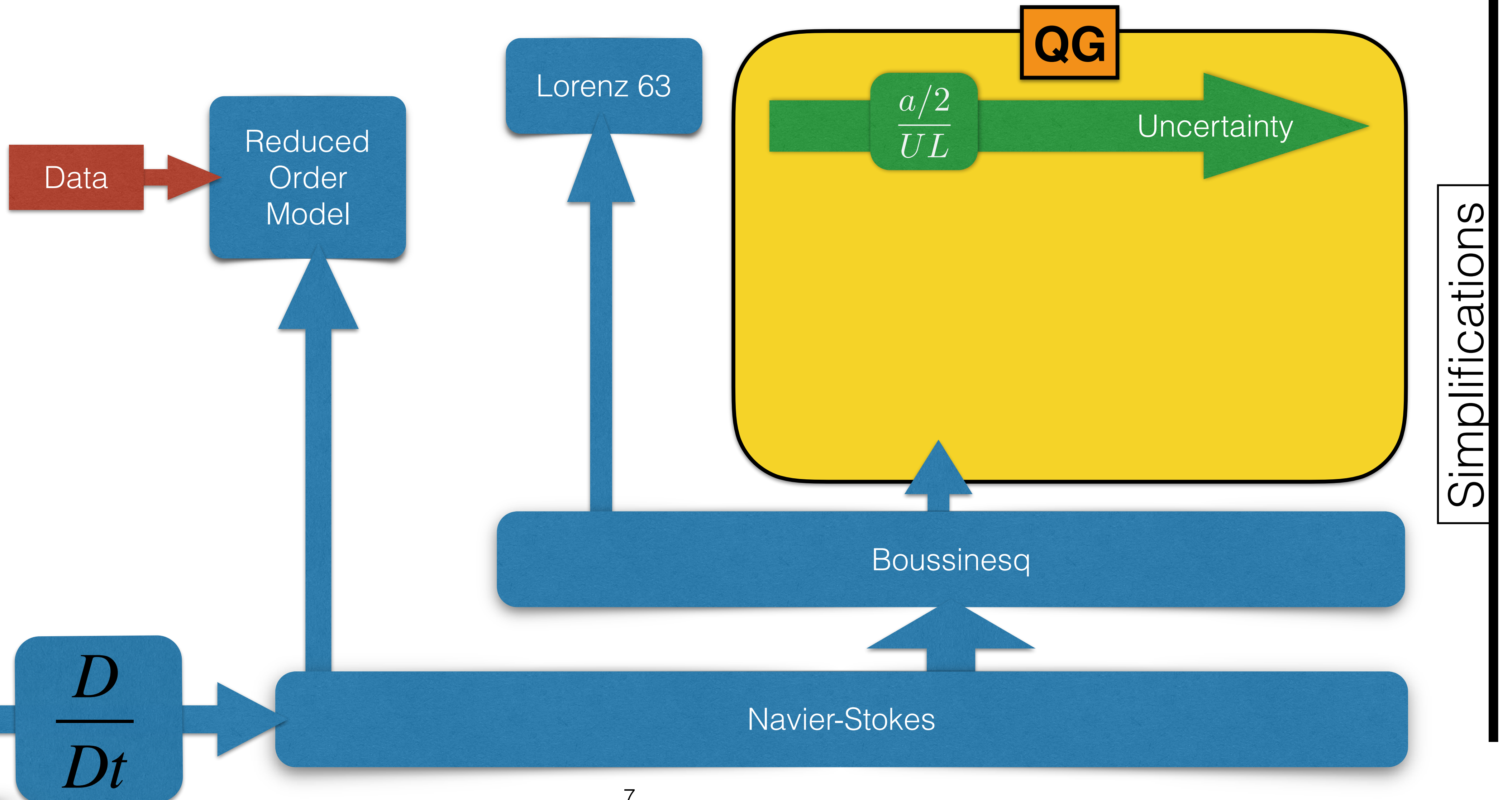
Conservations
 (mass, linear
 momentum, ...)



Derived random models

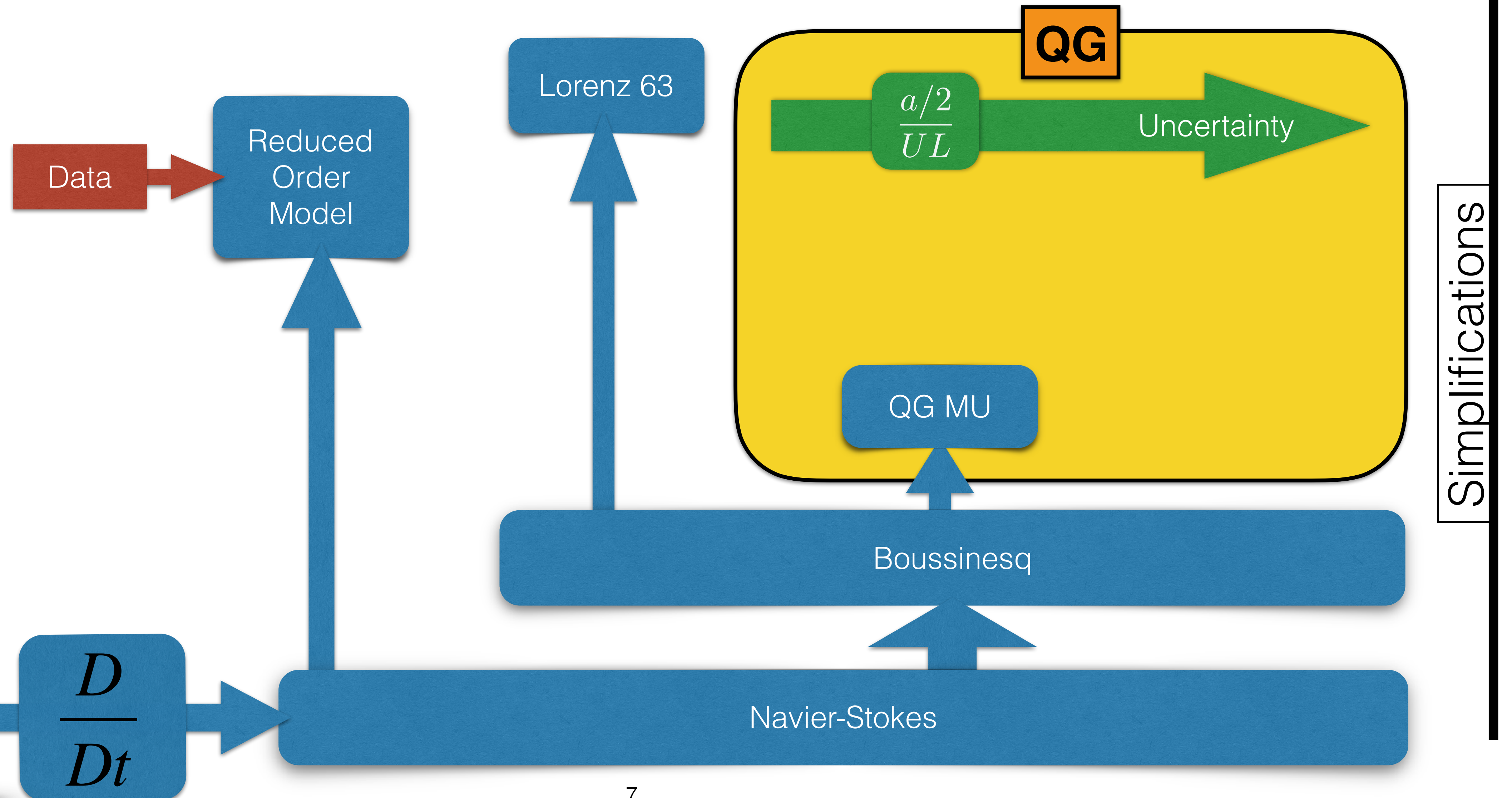
Large scales:
 \boldsymbol{w}
 Small scales:
 $\boldsymbol{\sigma} \dot{\boldsymbol{B}}$
 Variance
 tensor:
 $\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) =$
 $\frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Conservations
 (mass, linear
 momentum, ...)



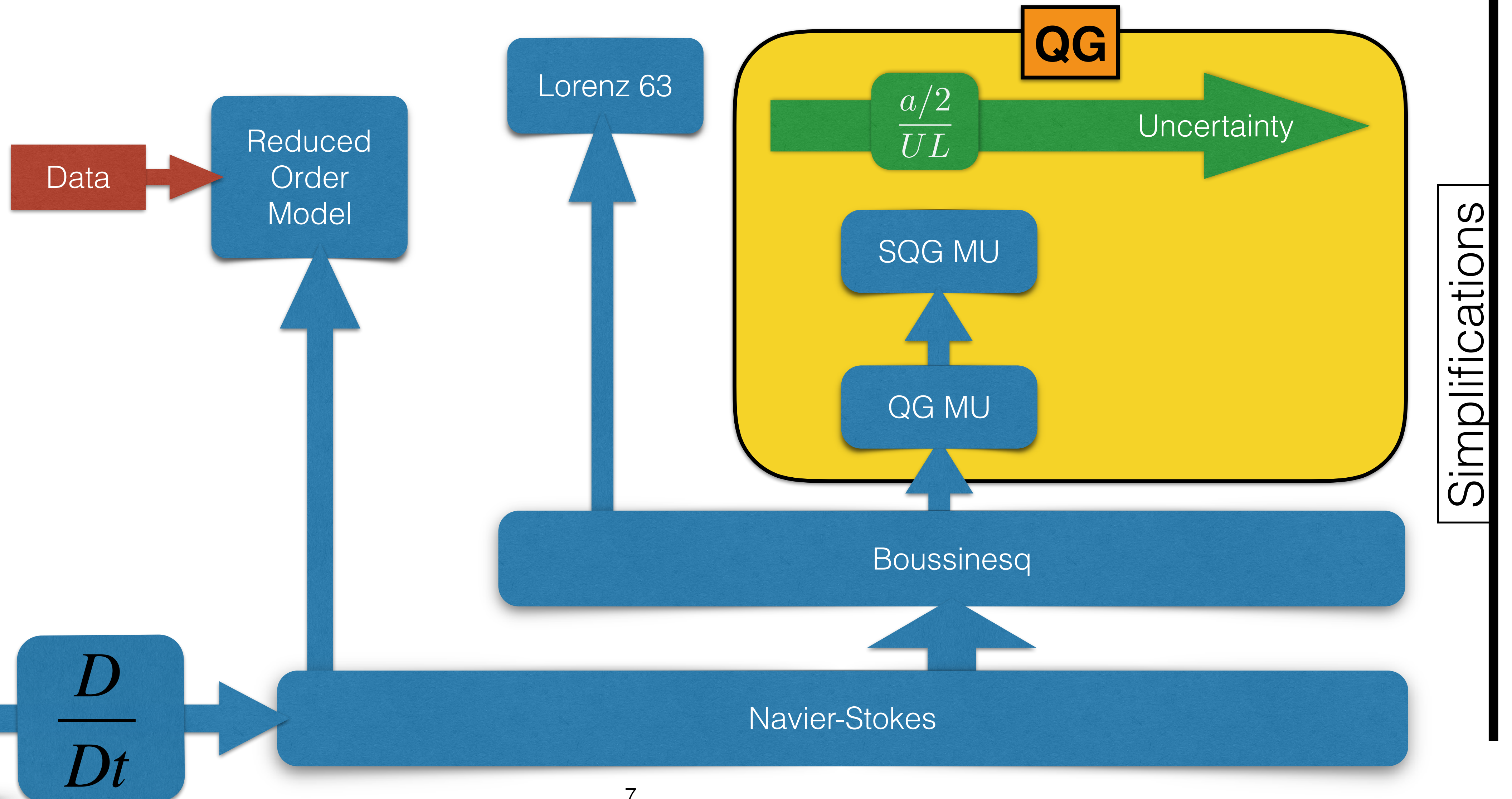
Derived random models

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance
 tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



Derived random models

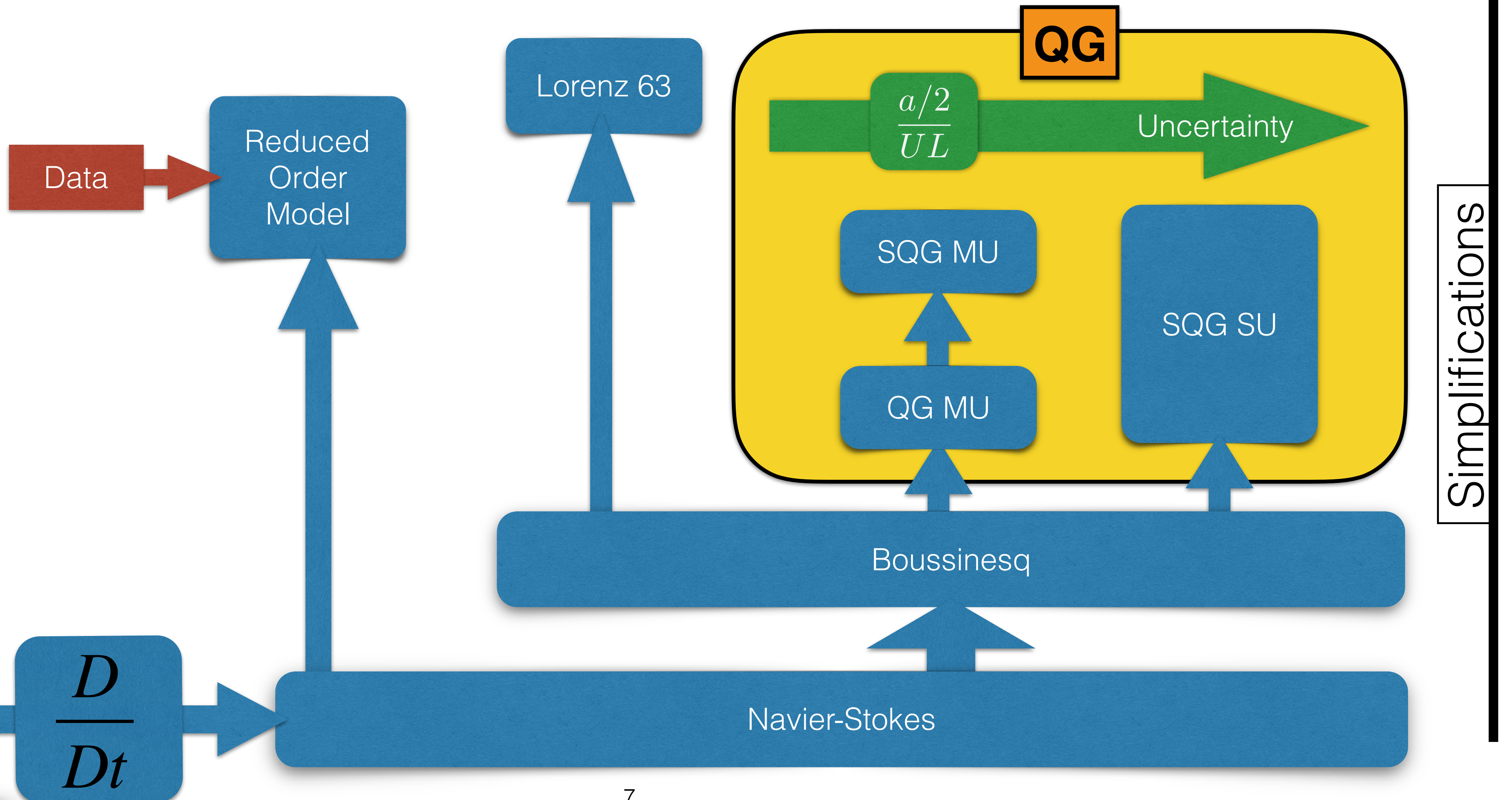
Large scales:
 \boldsymbol{w}
 Small scales:
 $\boldsymbol{\sigma} \dot{\boldsymbol{B}}$
 Variance tensor:
 $\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$



Derived random models

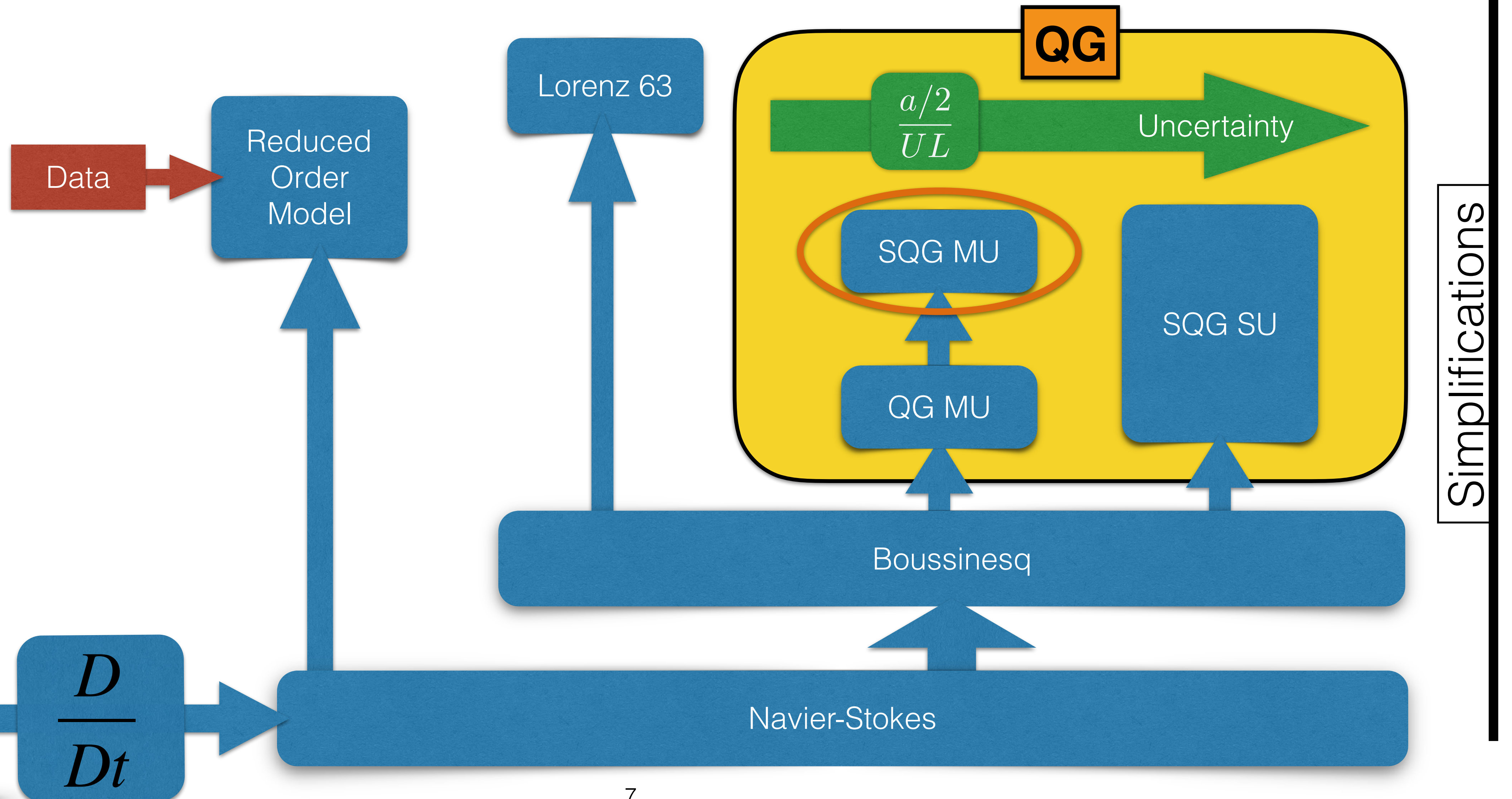
Large scales:
 \boldsymbol{w}
 Small scales:
 $\boldsymbol{\sigma} \dot{\boldsymbol{B}}$
 Variance tensor:
 $\boldsymbol{a} = \boldsymbol{a}(\boldsymbol{x}, \boldsymbol{x}) = \frac{\mathbb{E}\{\boldsymbol{\sigma} d\boldsymbol{B} (\boldsymbol{\sigma} d\boldsymbol{B})^T\}}{dt}$

Conservations
 (mass, linear
 momentum, ...)



Derived random models

Large scales:
 w
 Small scales:
 $\sigma \dot{B}$
 Variance tensor:
 $a = a(x, x) = \frac{\mathbb{E}\{\sigma dB (\sigma dB)^T\}}{dt}$



Part II

SQG under Moderate Uncertainty

SQG MU

Code available online

$t = 17$ day

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

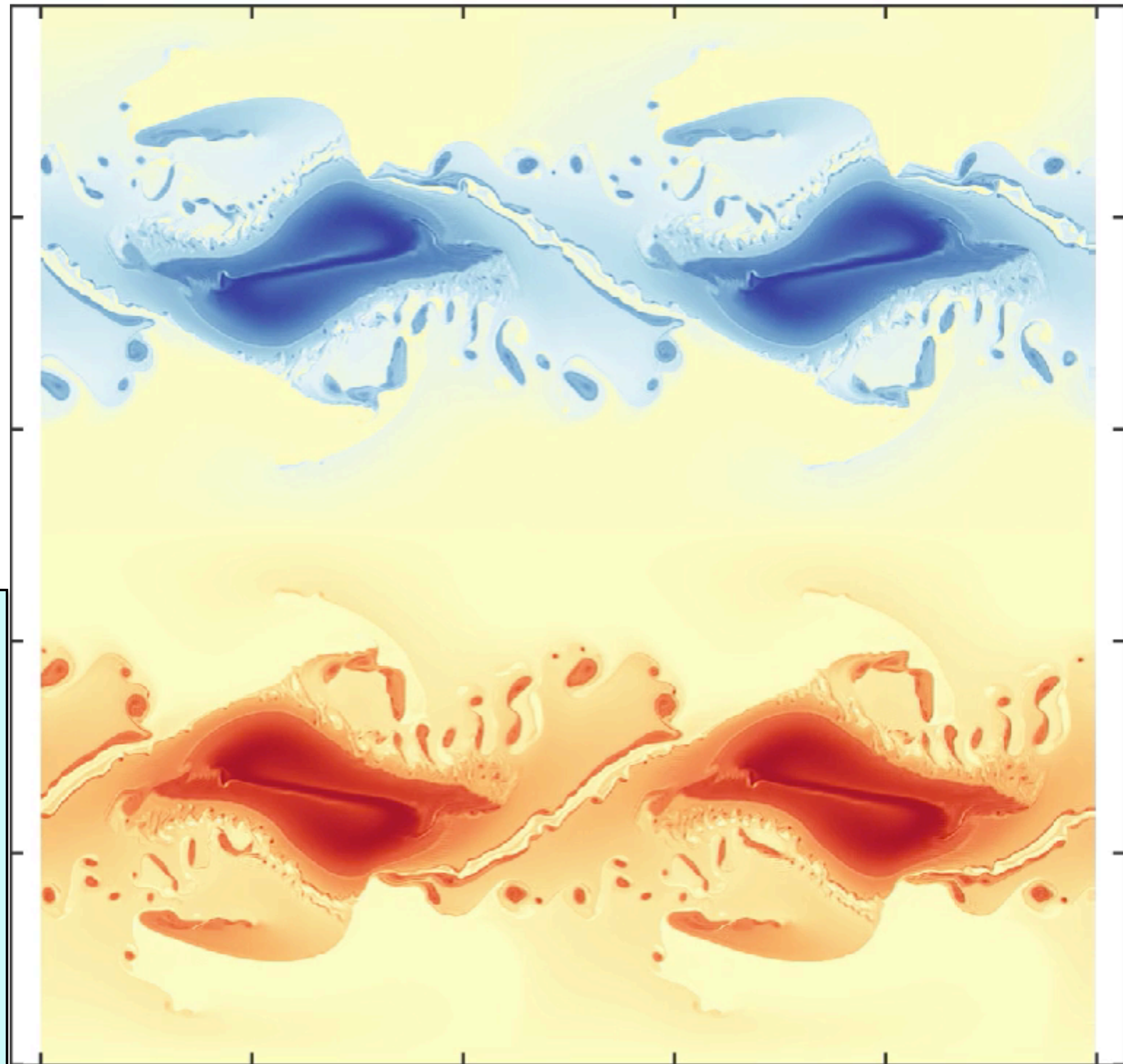
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

Reference flow:

deterministic

SQG

1024 x 1024



$t = 17$ day

SQG

$$\frac{Db}{Dt} = -\alpha_{HV} \Delta^4 b \quad \text{Hyper-viscosity}$$

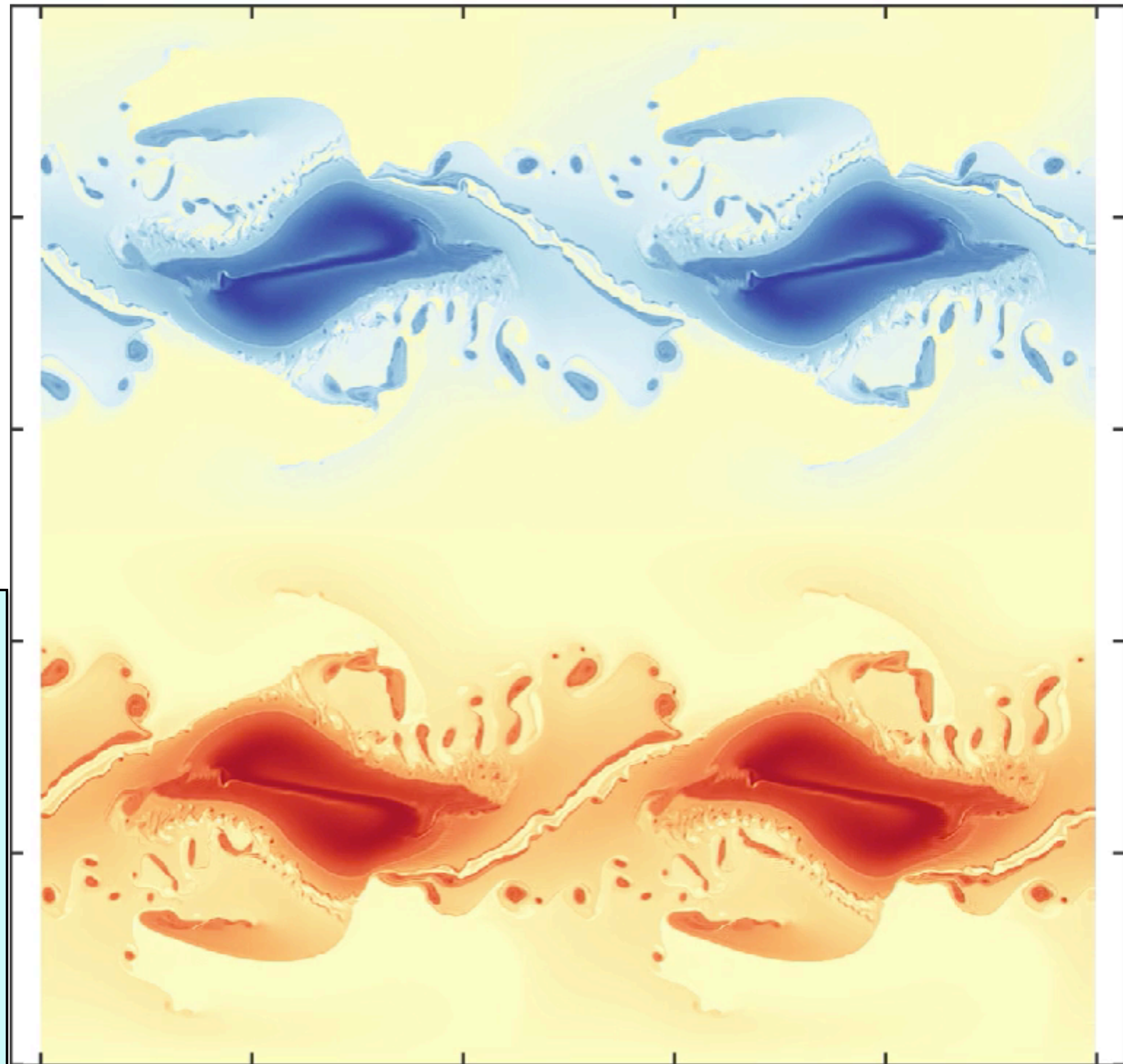
$$\mathbf{u} = \left(\text{cst.} \nabla^\perp \Delta^{-\frac{1}{2}} \right) b$$

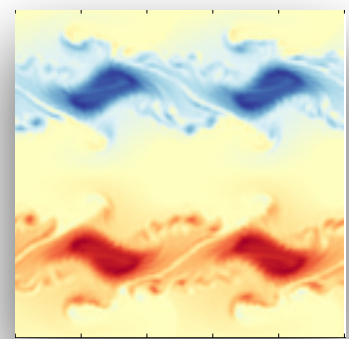
Reference flow:

deterministic

SQG

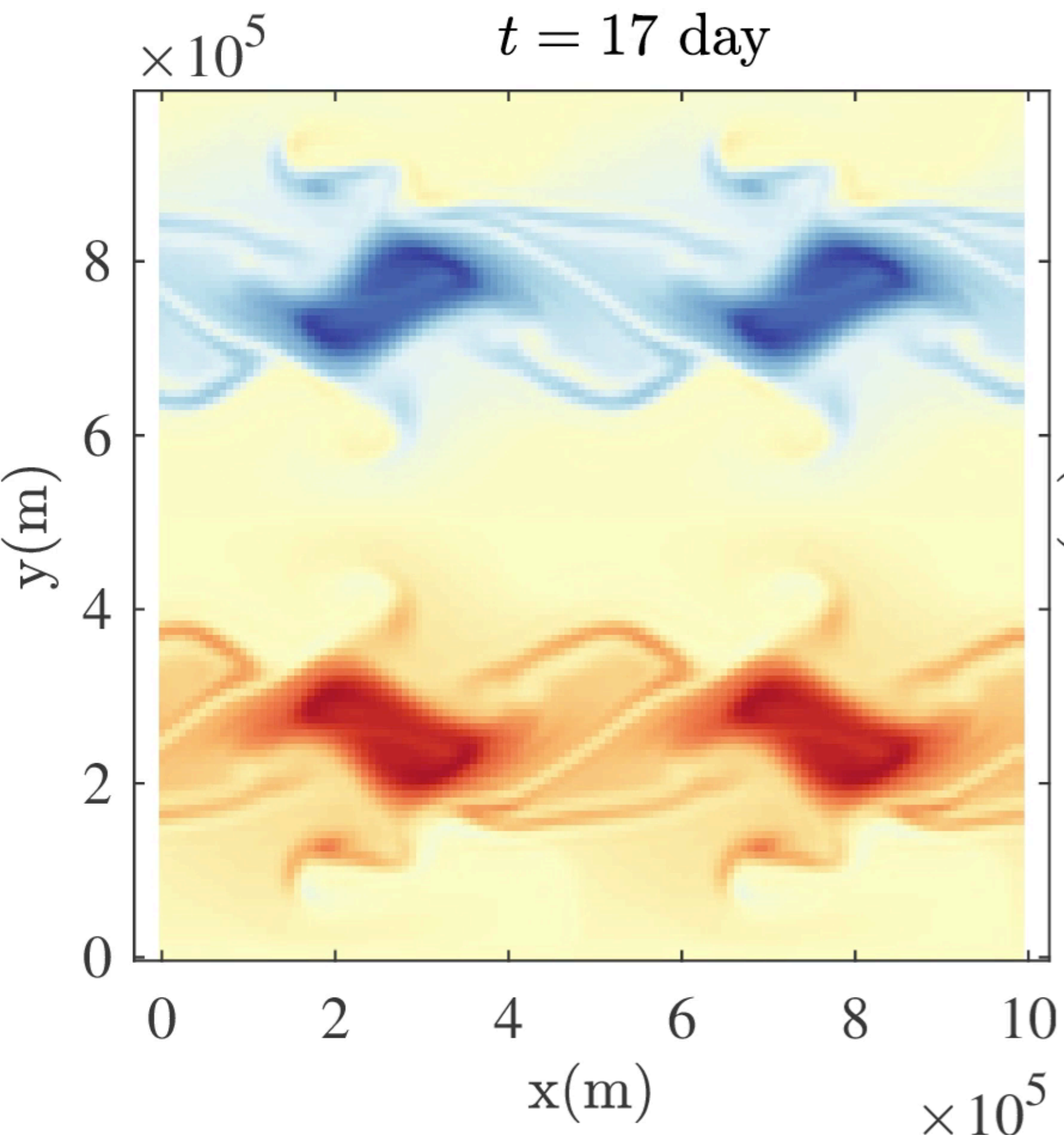
1024 x 1024



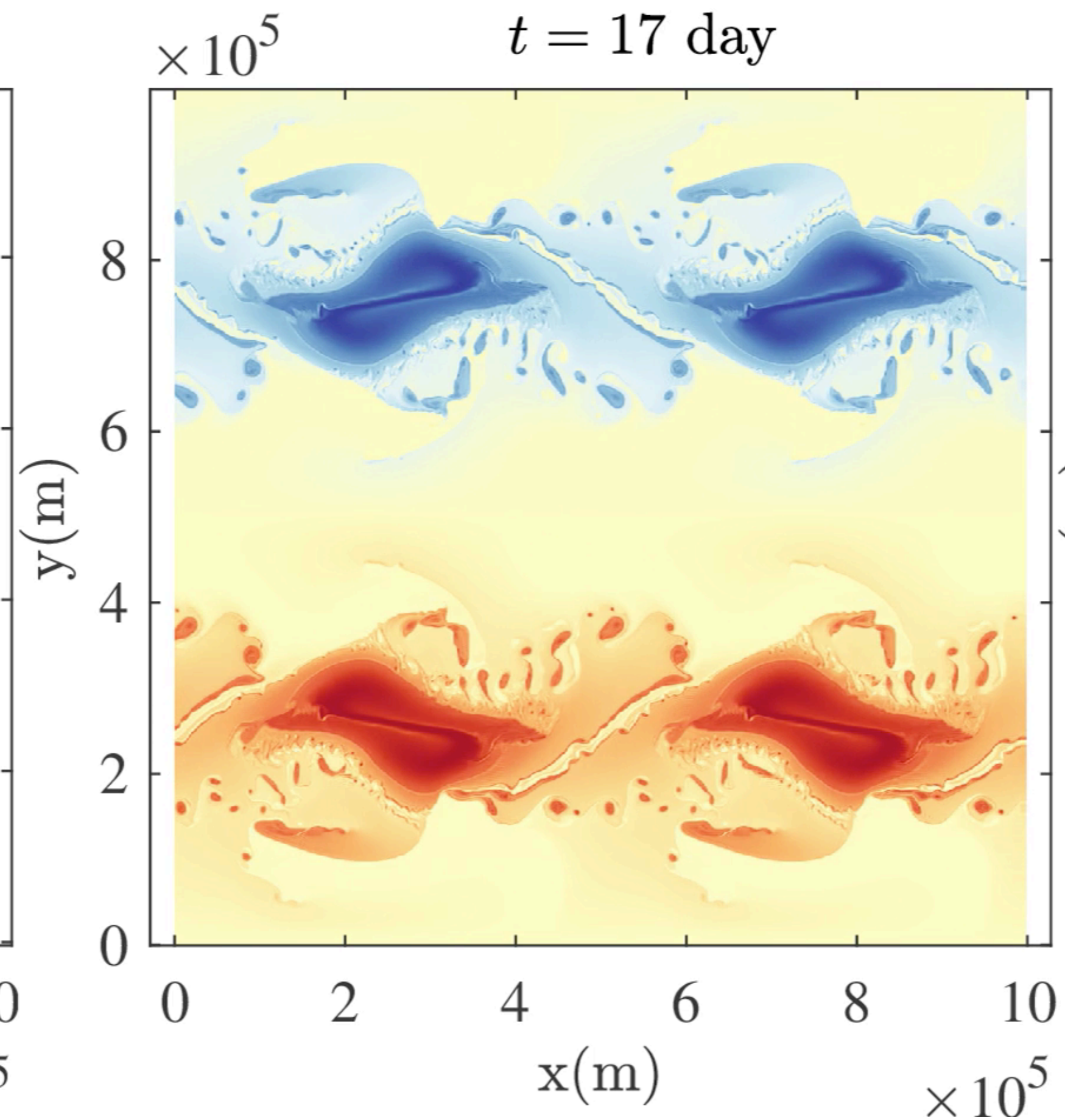


One realization : Stochastic destabilization

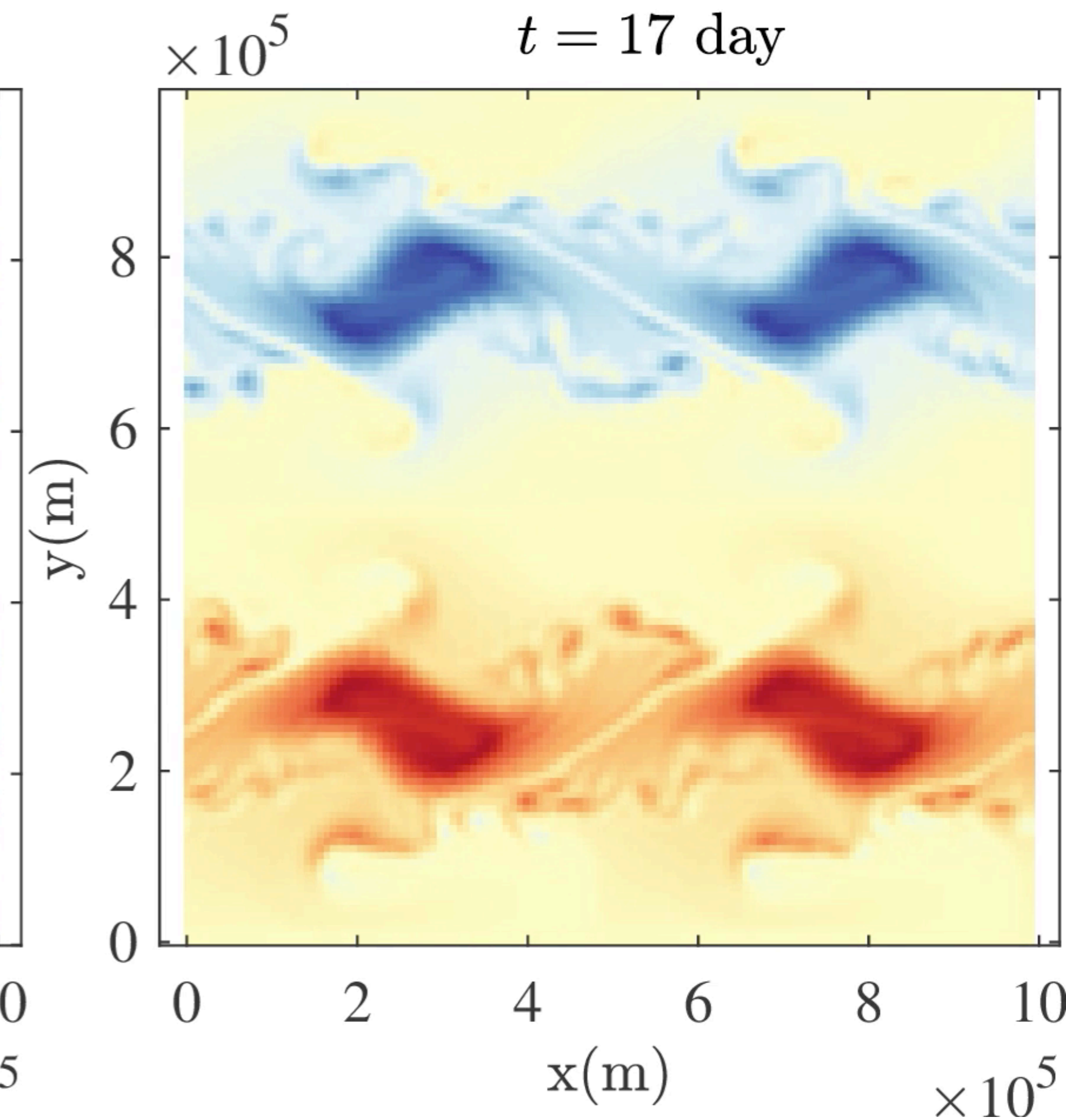
Deterministic 128 x 128

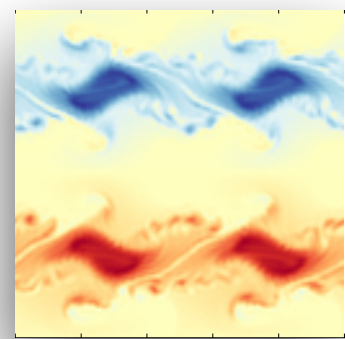


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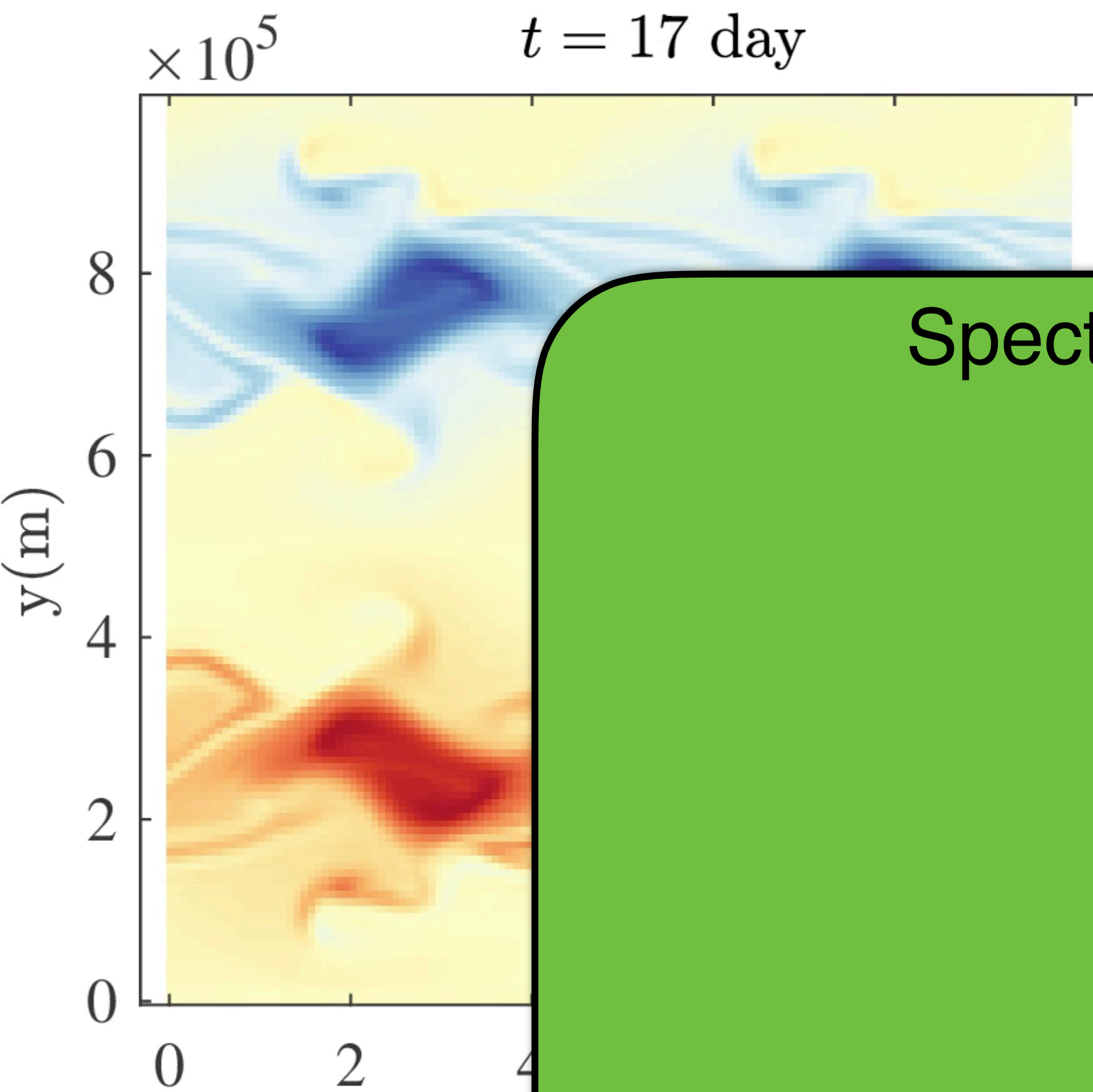
Location Uncertainty 128 x 128



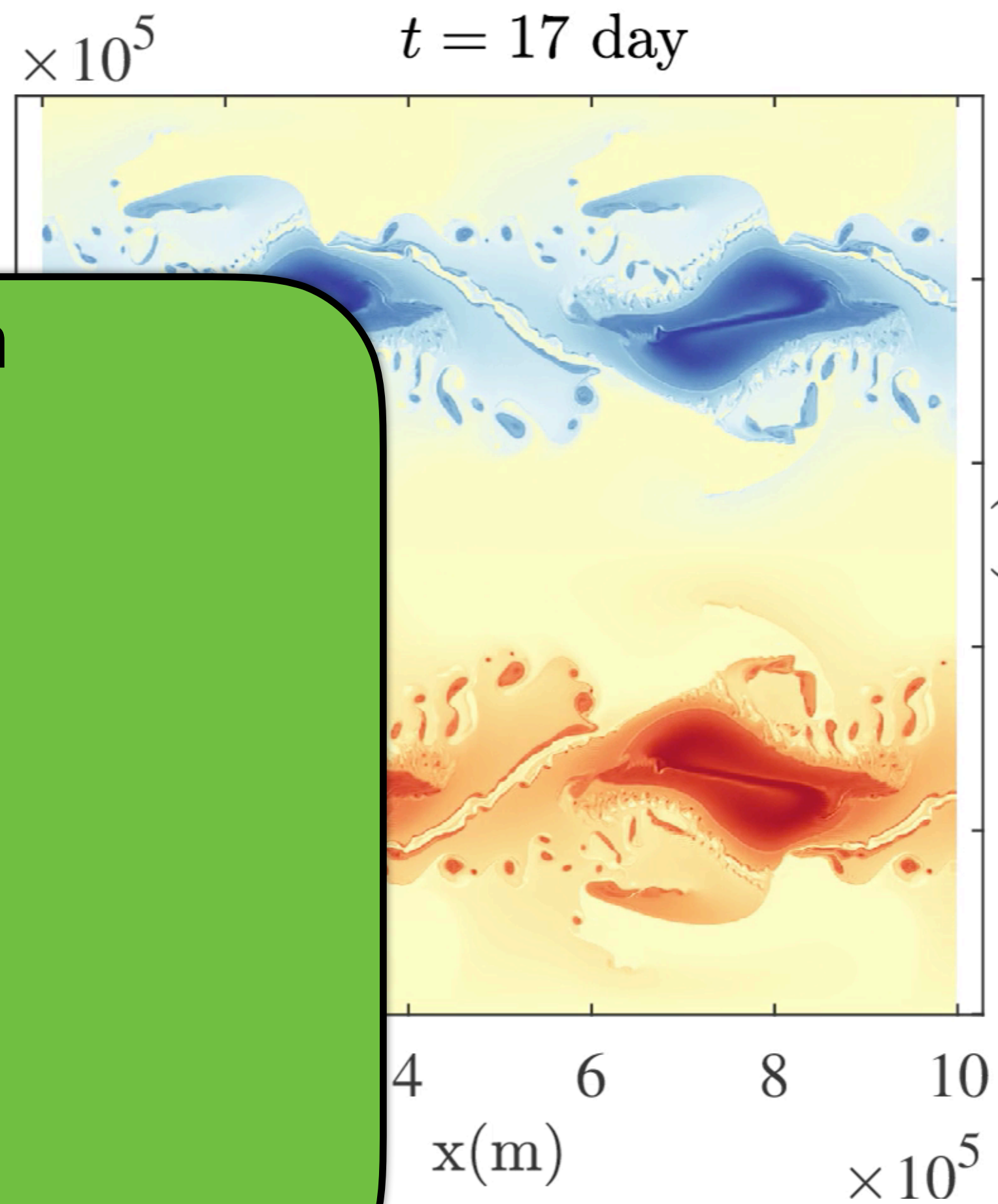


One realization : Stochastic destabilization

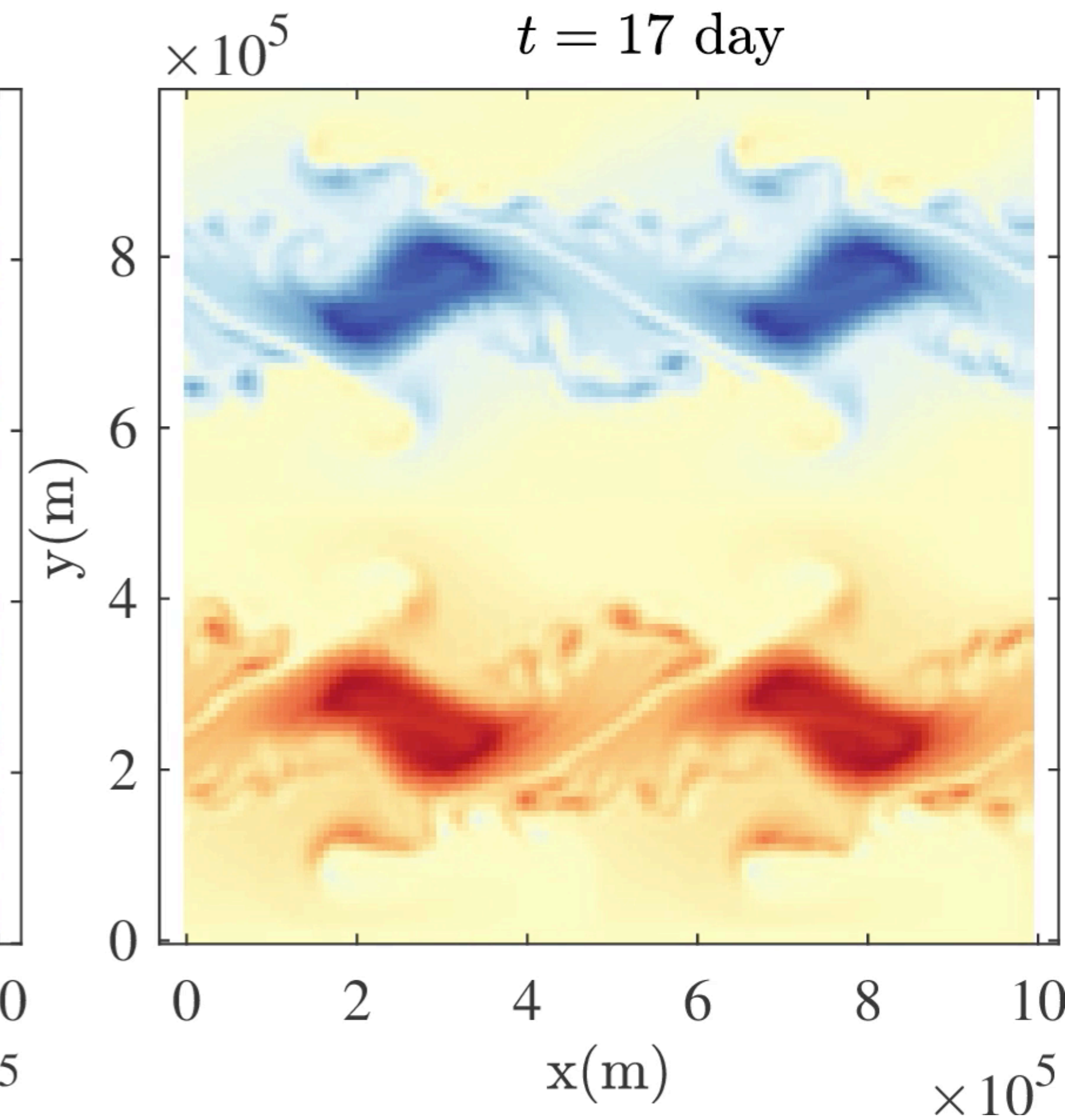
Deterministic 128 x 128

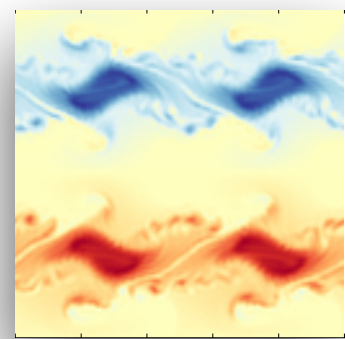


Deterministic 1024 x 1024



Location Uncertainty 128 x 128



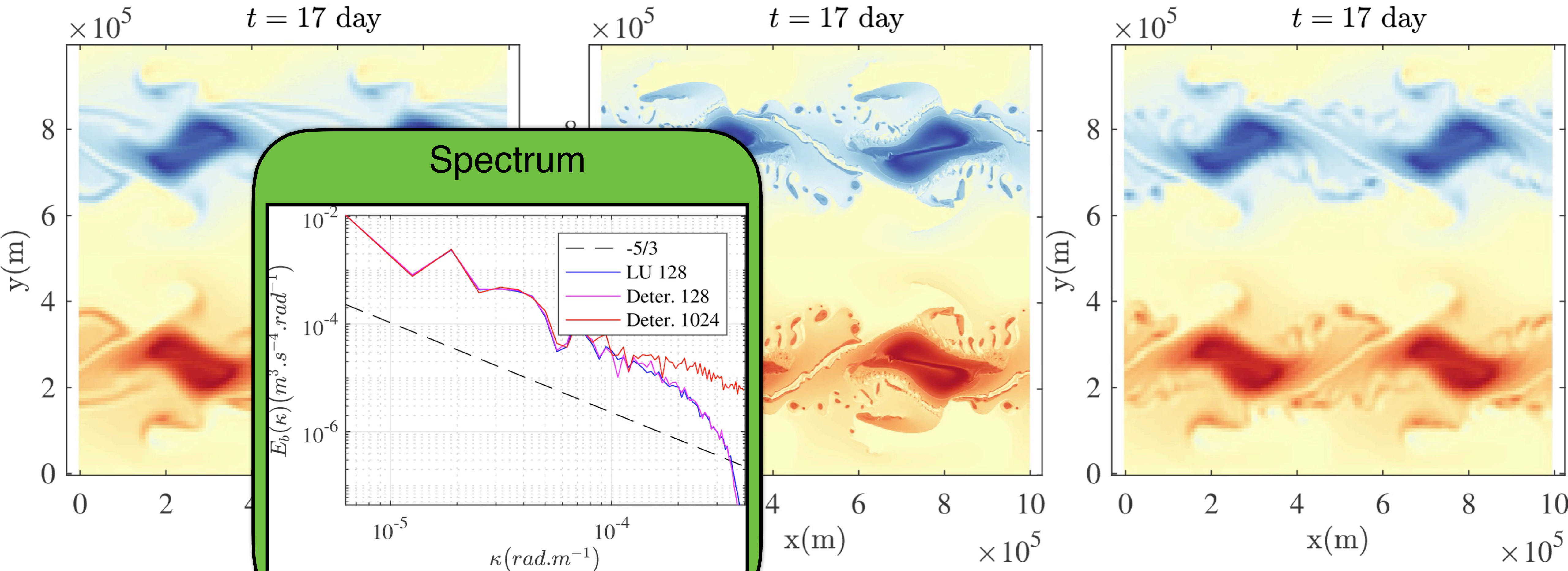


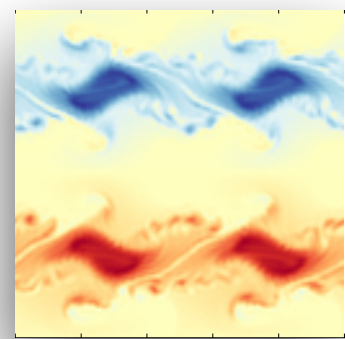
One realization : Stochastic destabilization

Deterministic 128 x 128

Deterministic 1024 x 1024

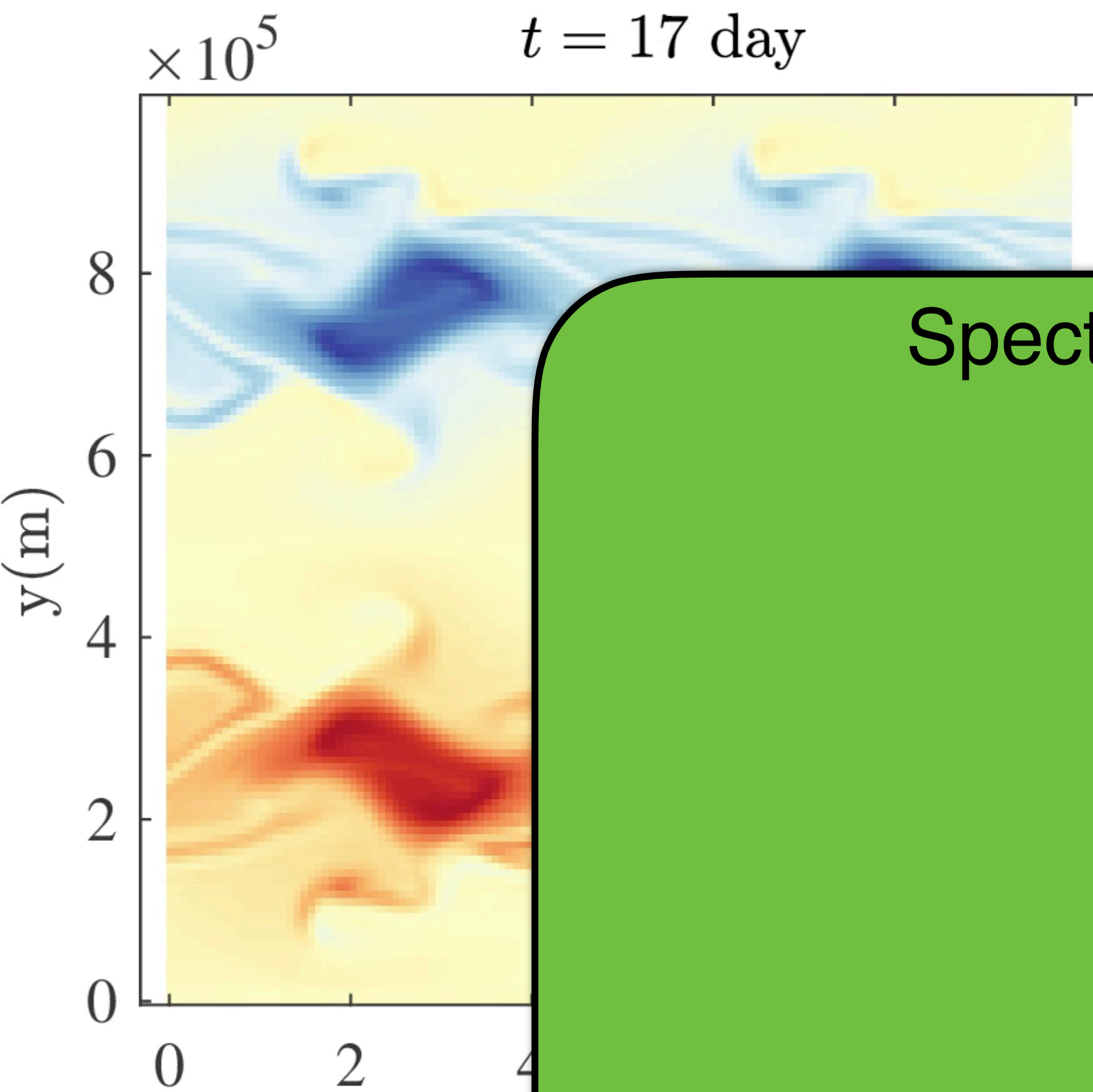
Location Uncertainty 128 x 128



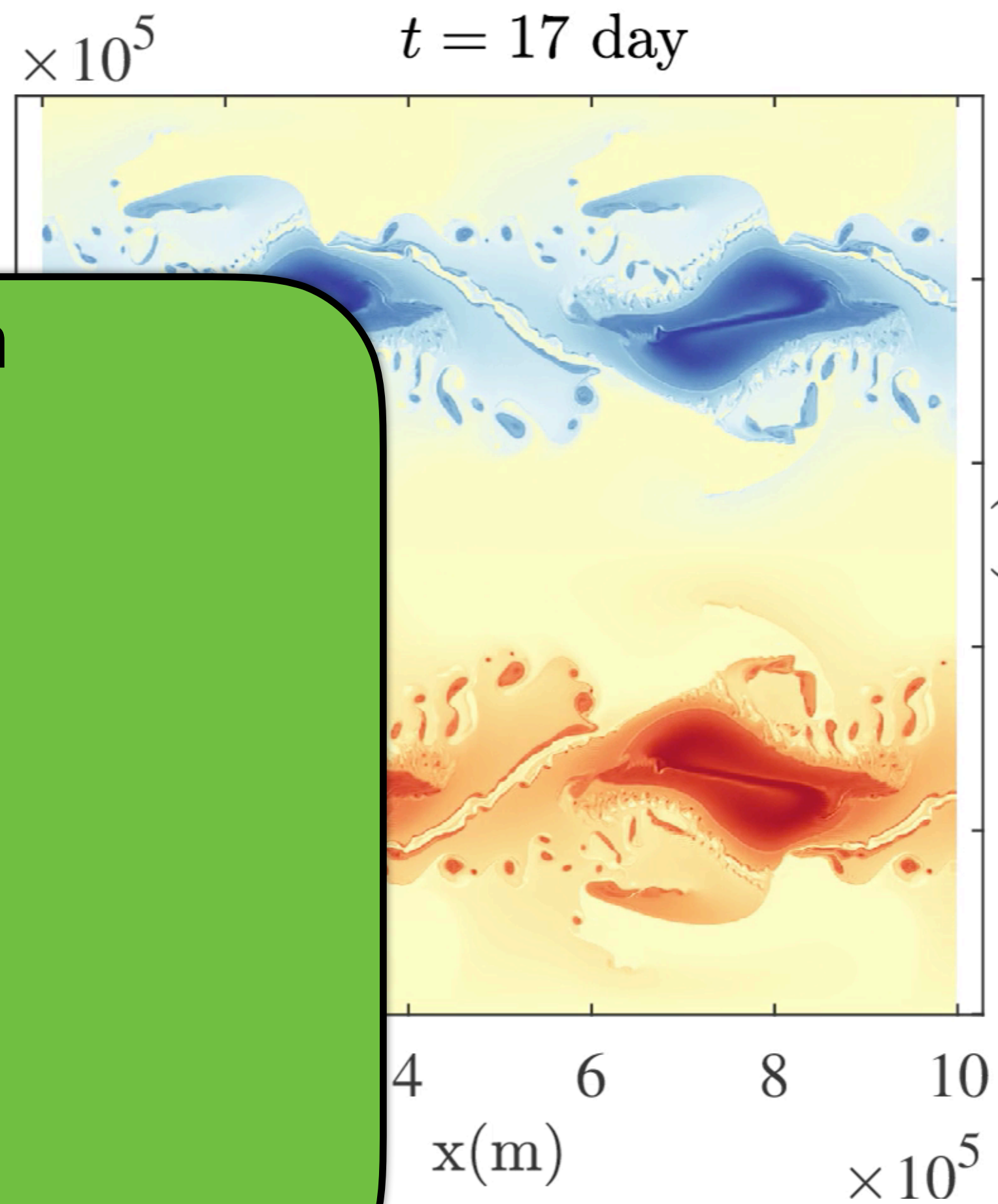


One realization : Stochastic destabilization

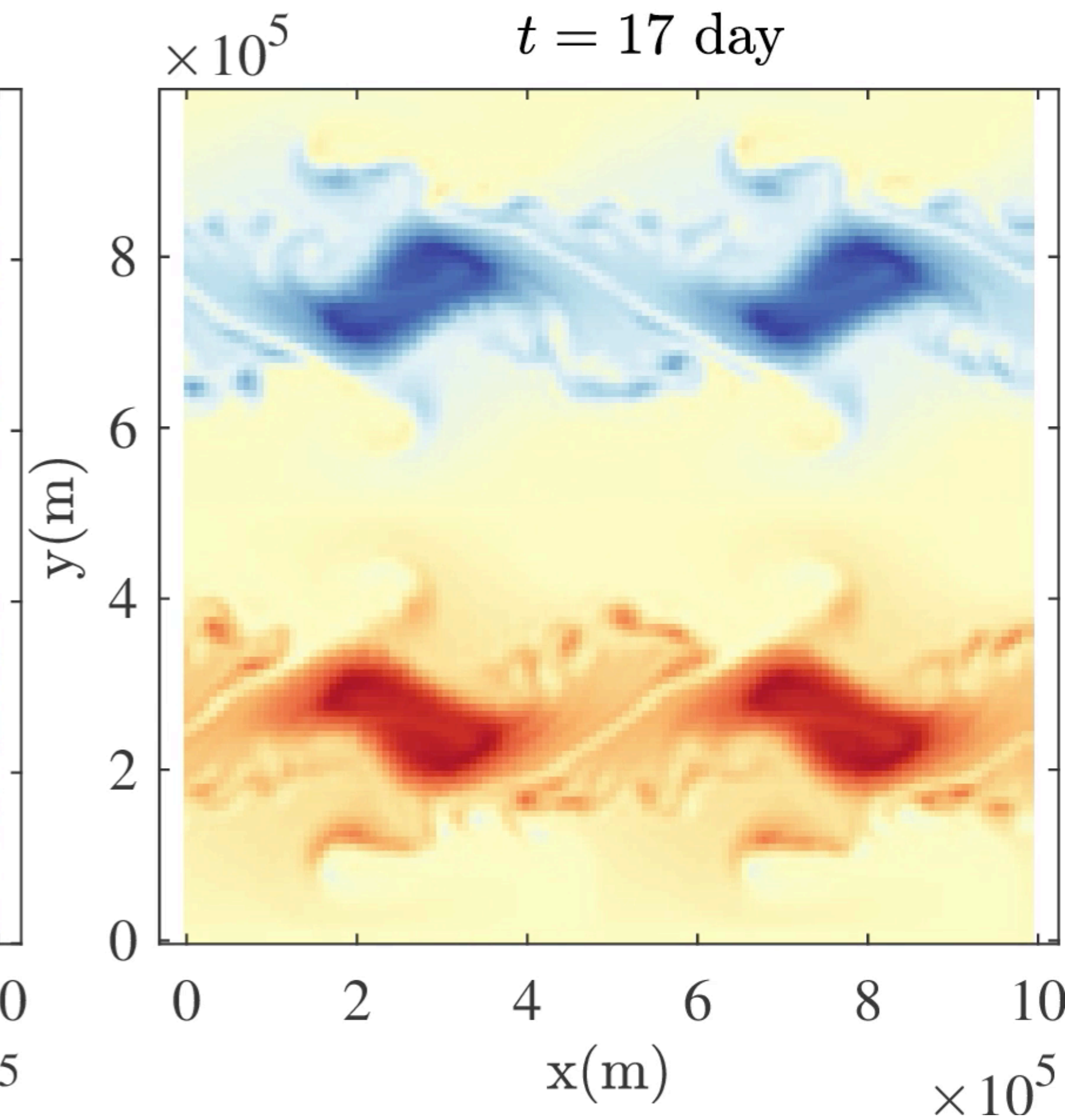
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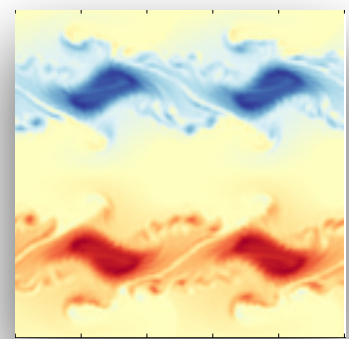


Deterministic 1024 x 1024



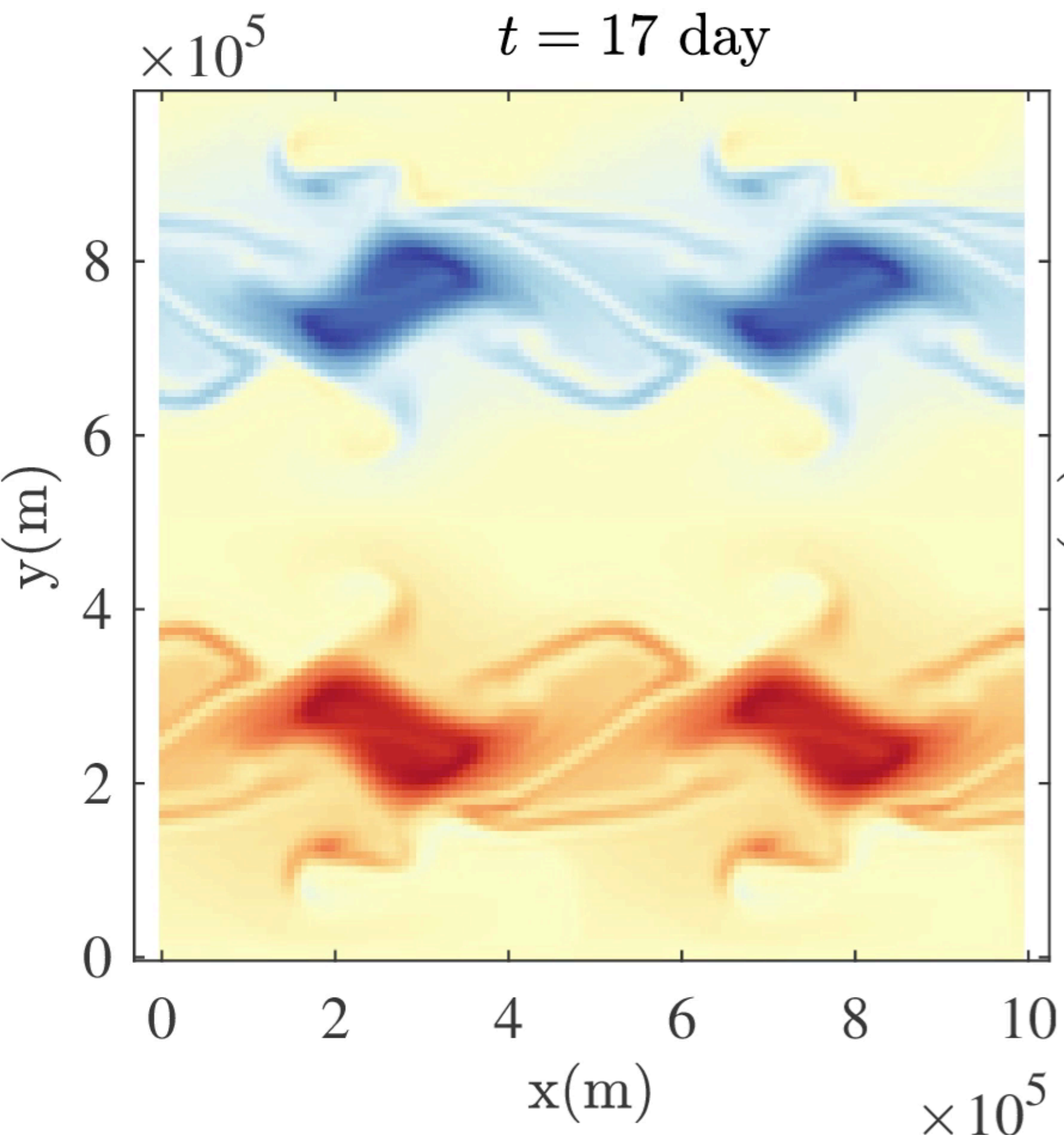
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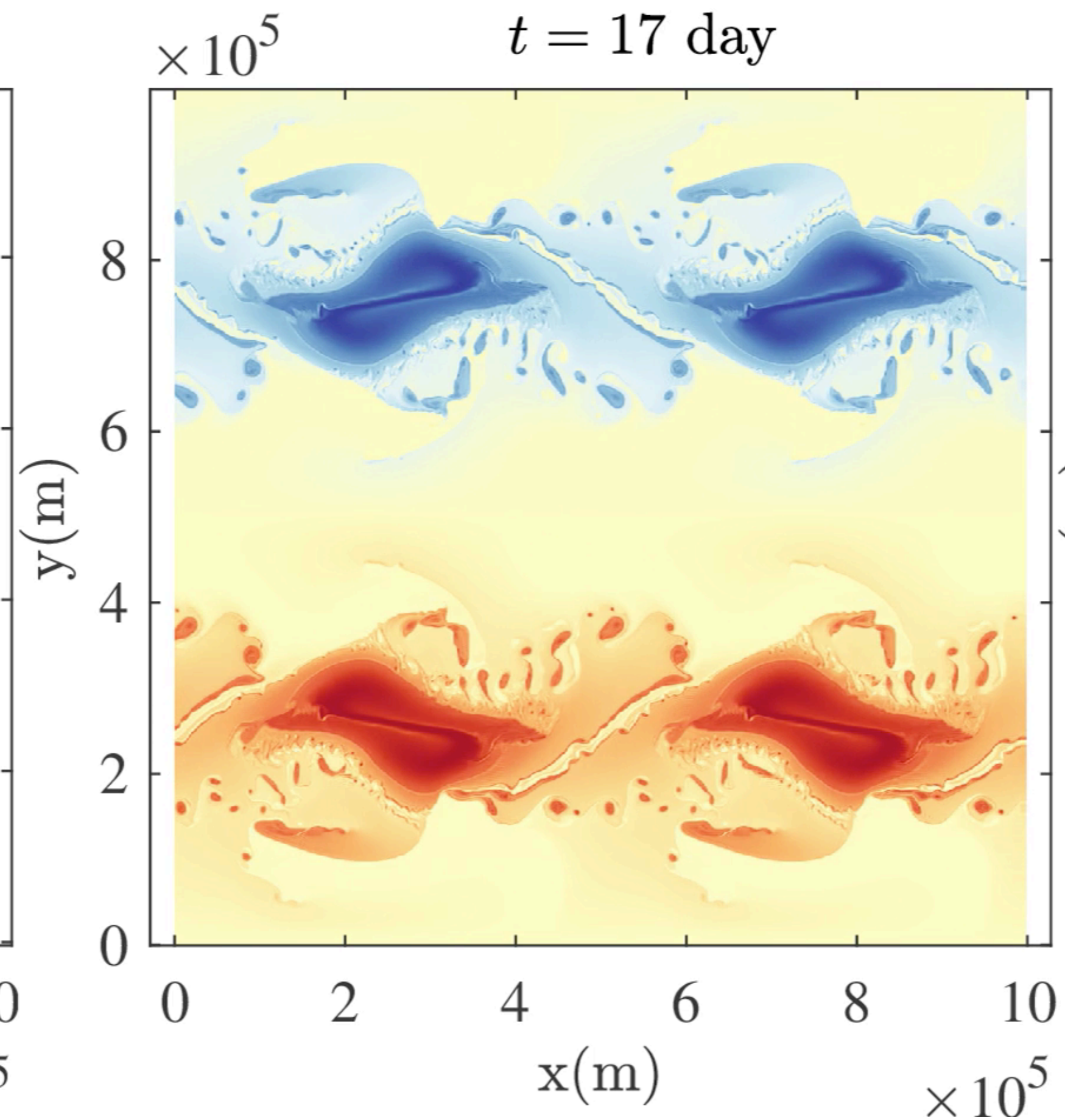


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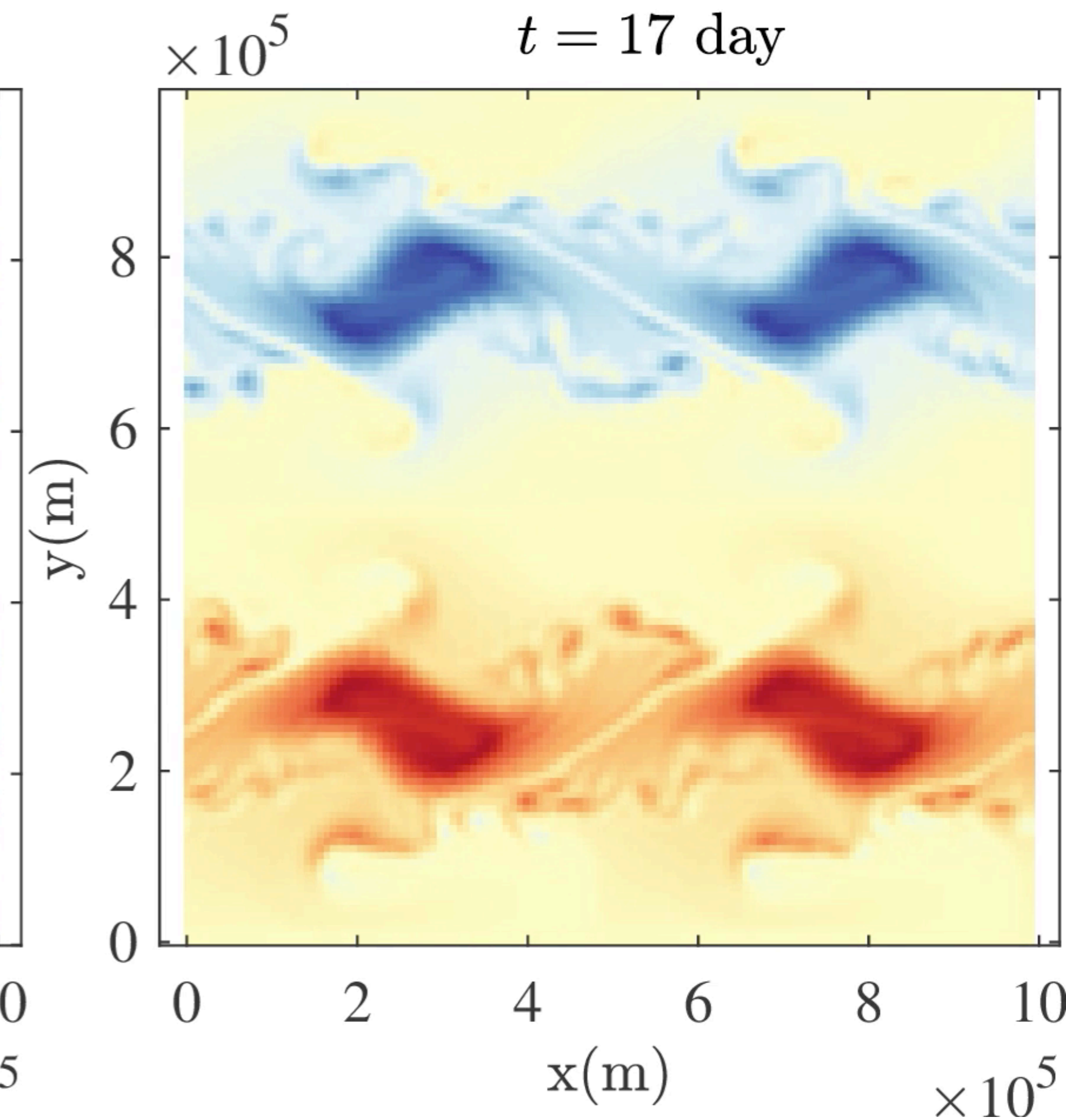
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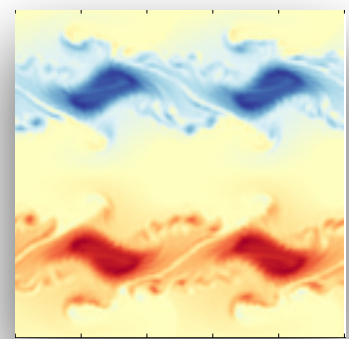


Deterministic 1024 x 1024



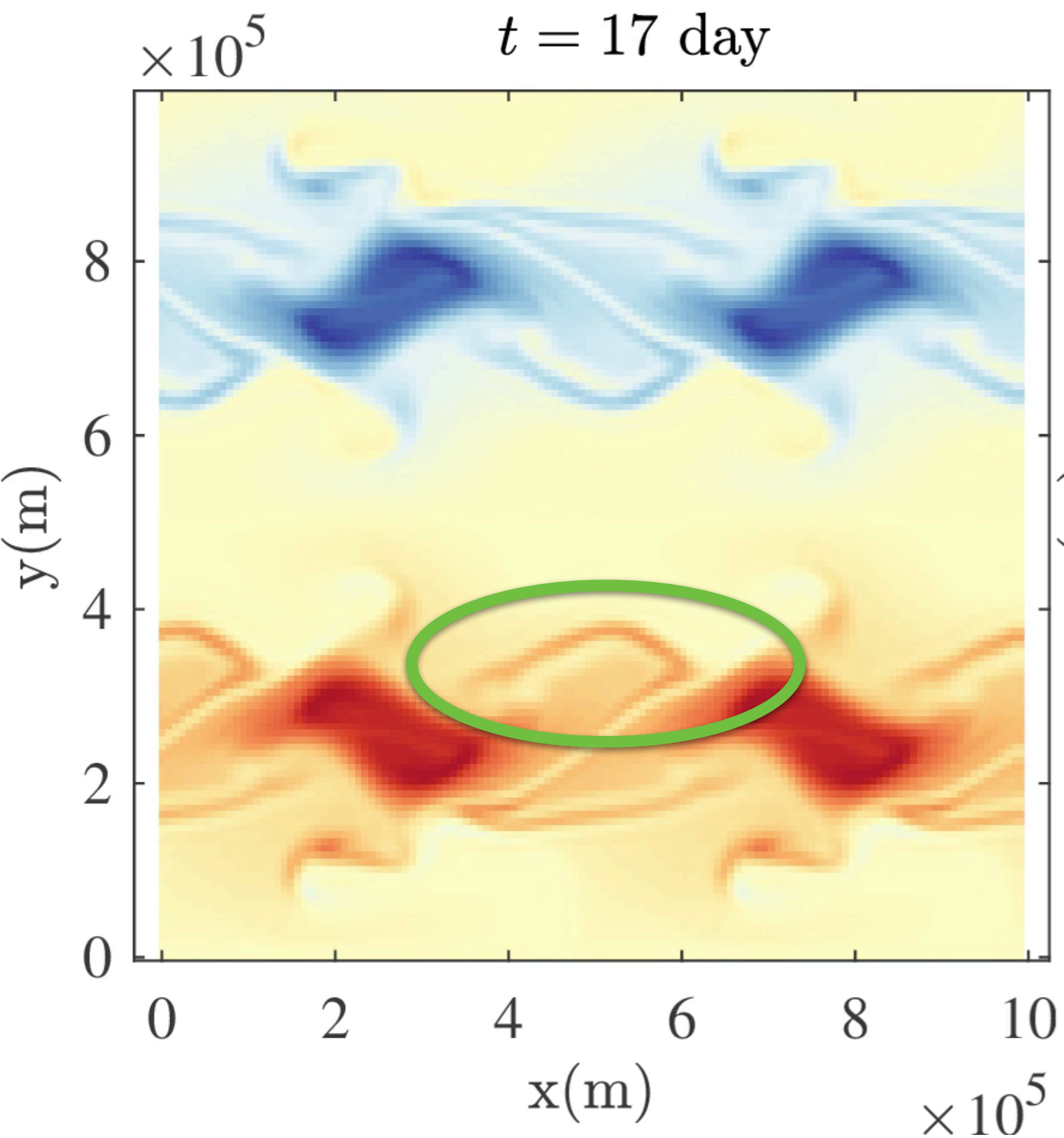
Location Uncertainty 128 x 128



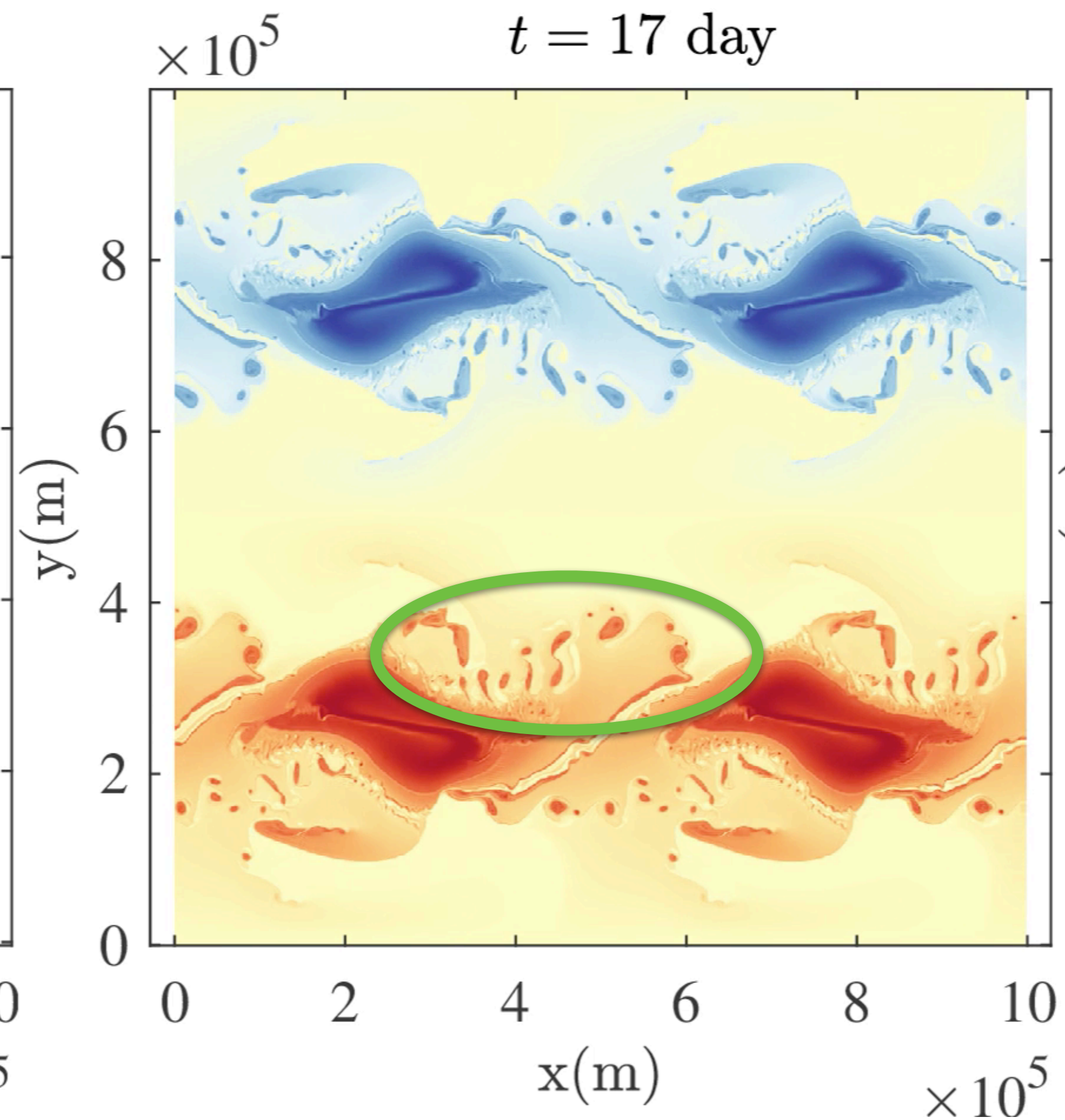


One realization : Stochastic destabilization

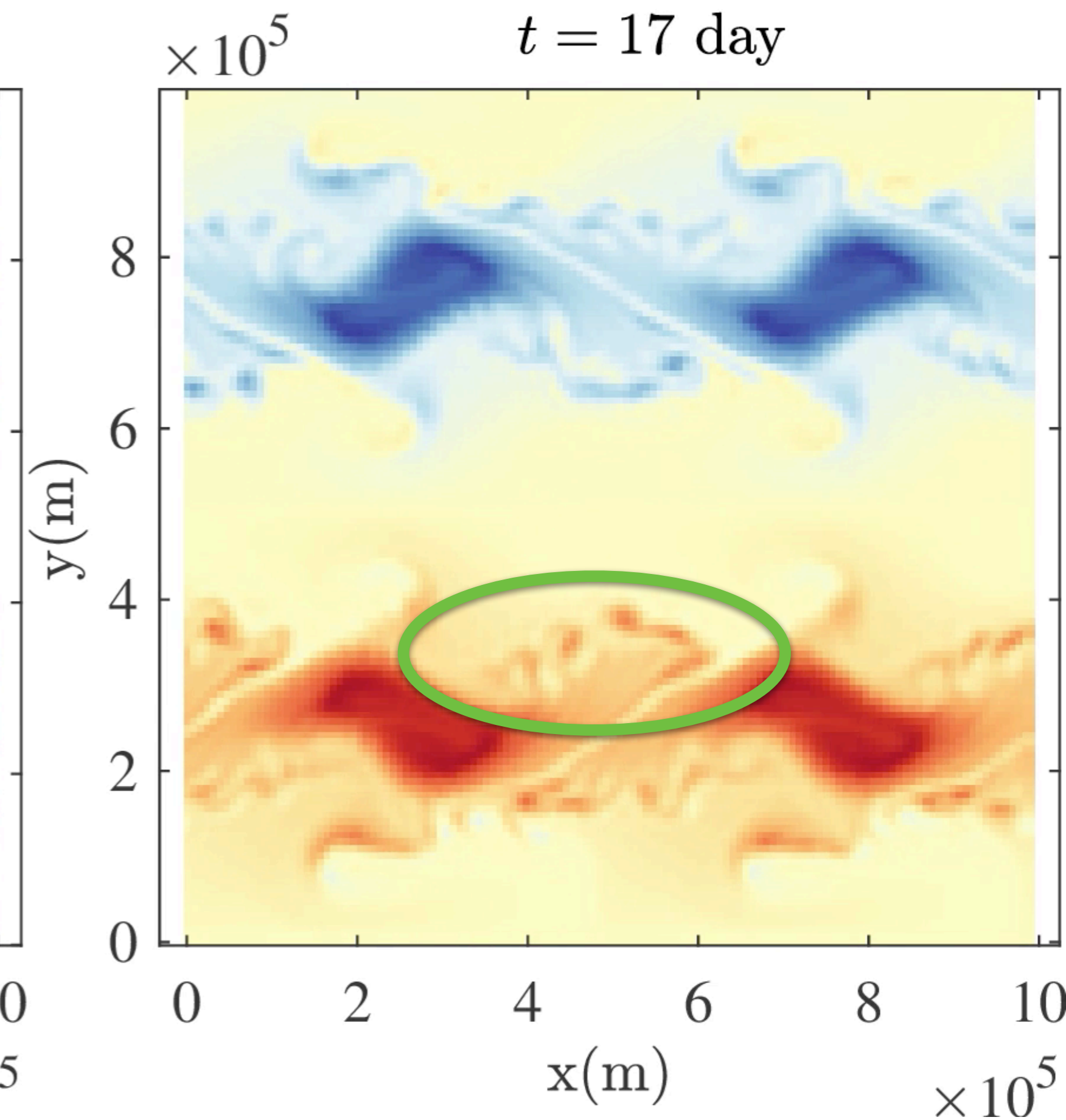
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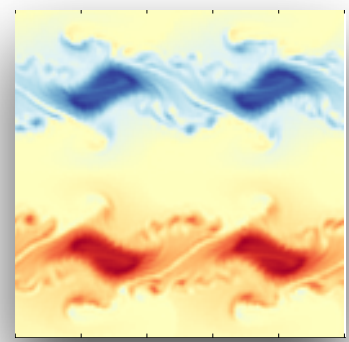


Deterministic 1024 x 1024



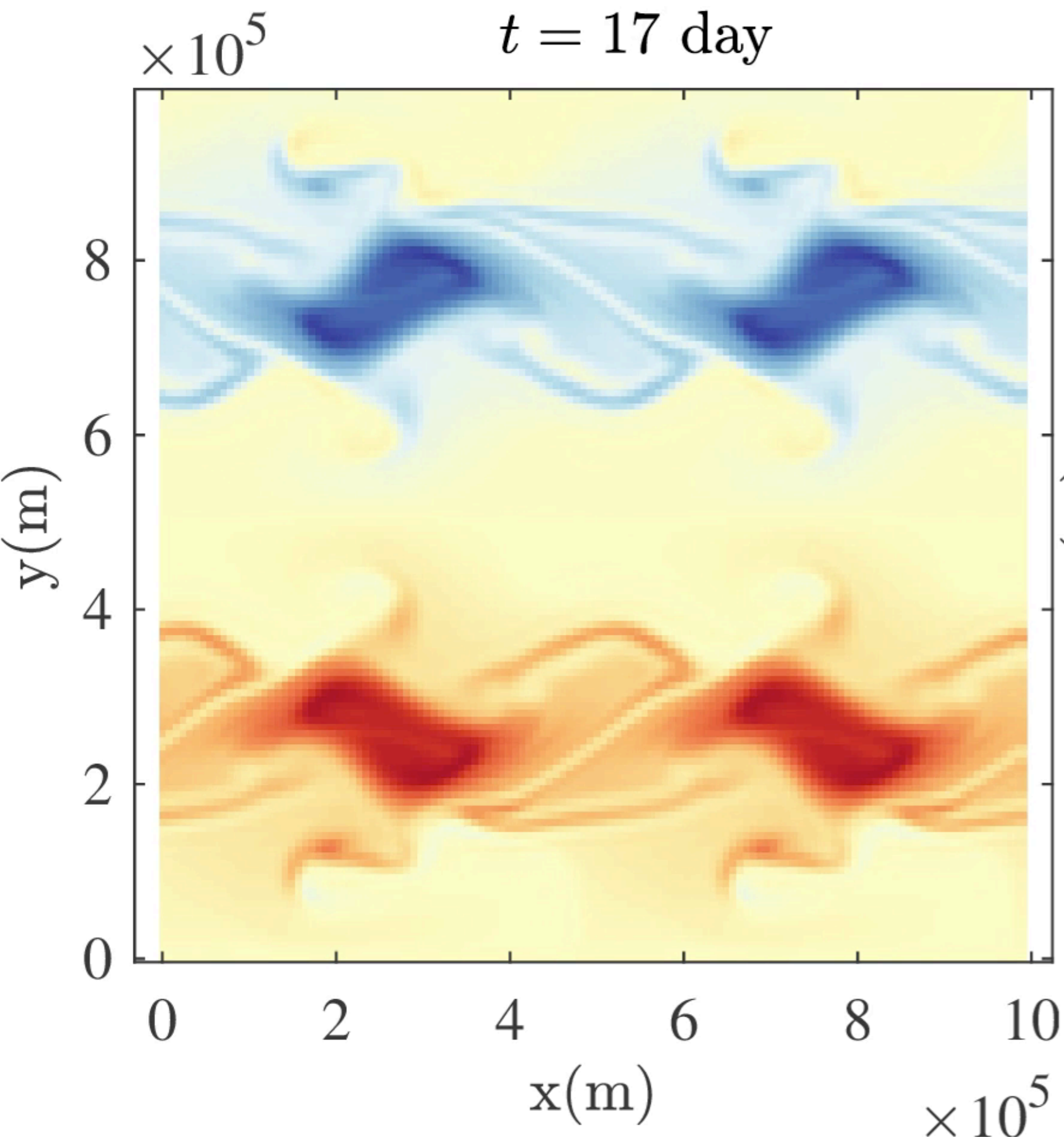
Location Uncertainty 128 x 128



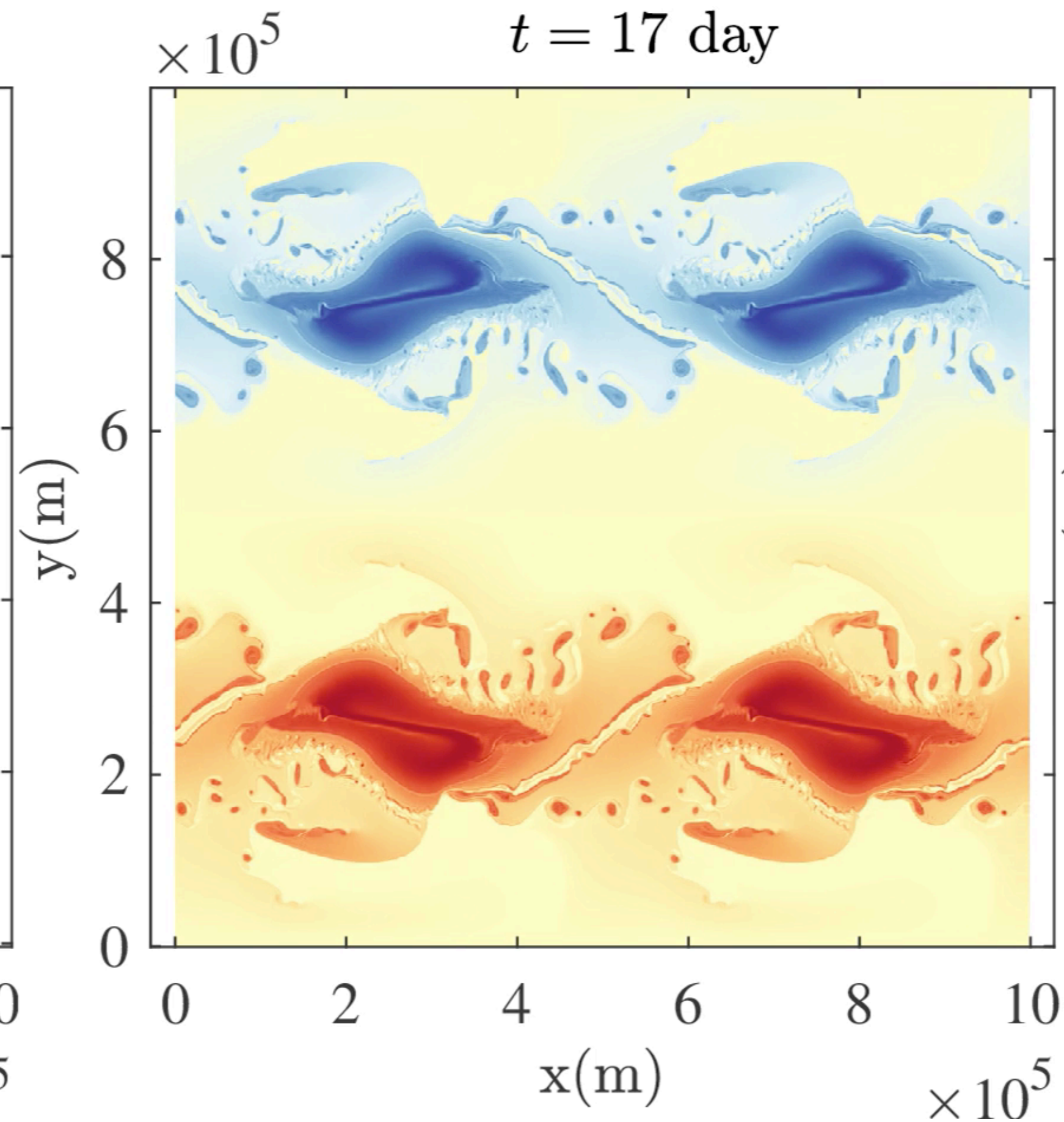


One realization : Stochastic destabilization

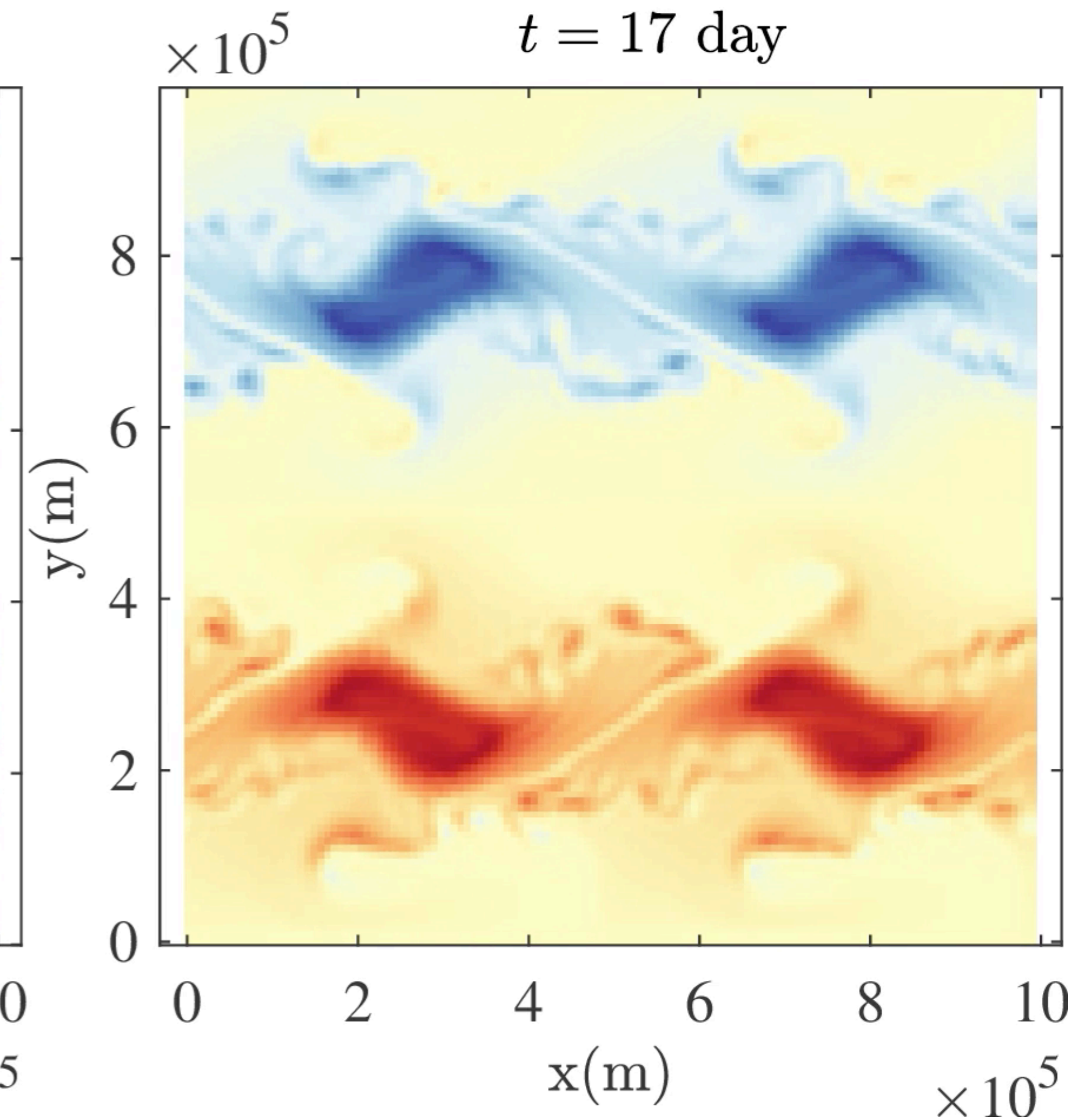
Deterministic 128 x 128

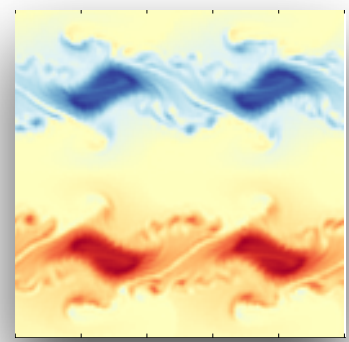


Deterministic 1024 x 1024



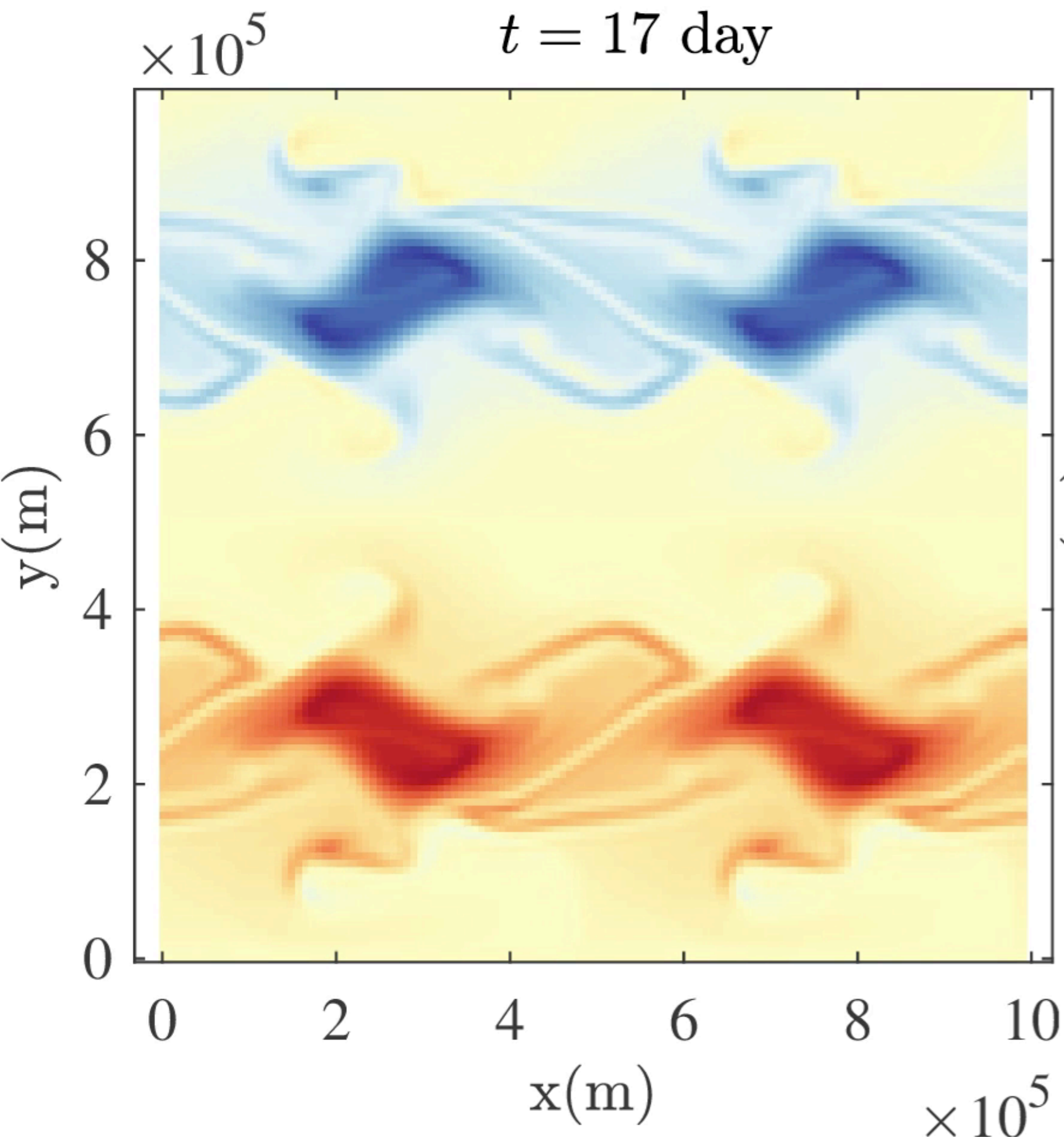
Location Uncertainty 128 x 128



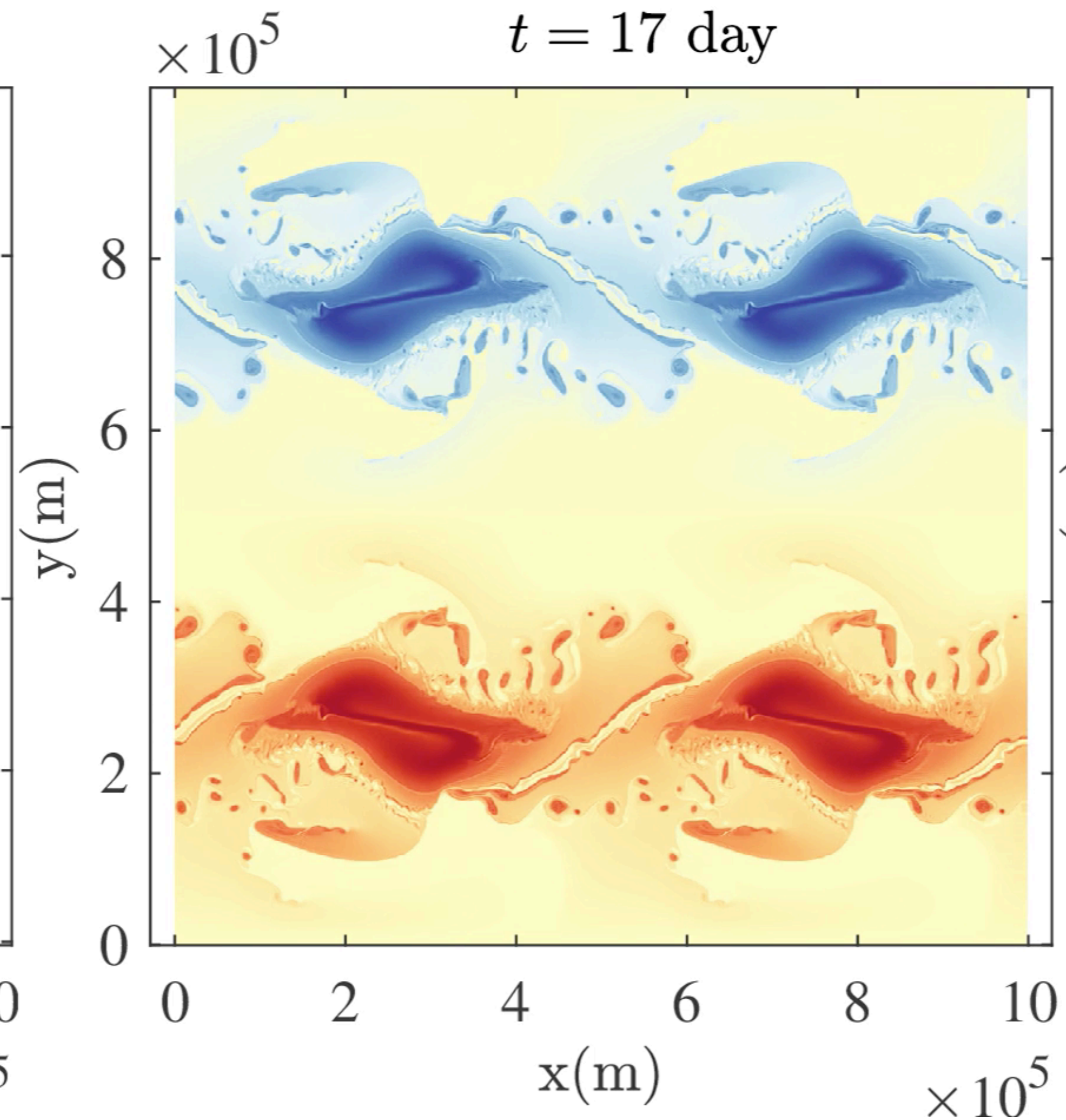


One realization : Stochastic destabilization

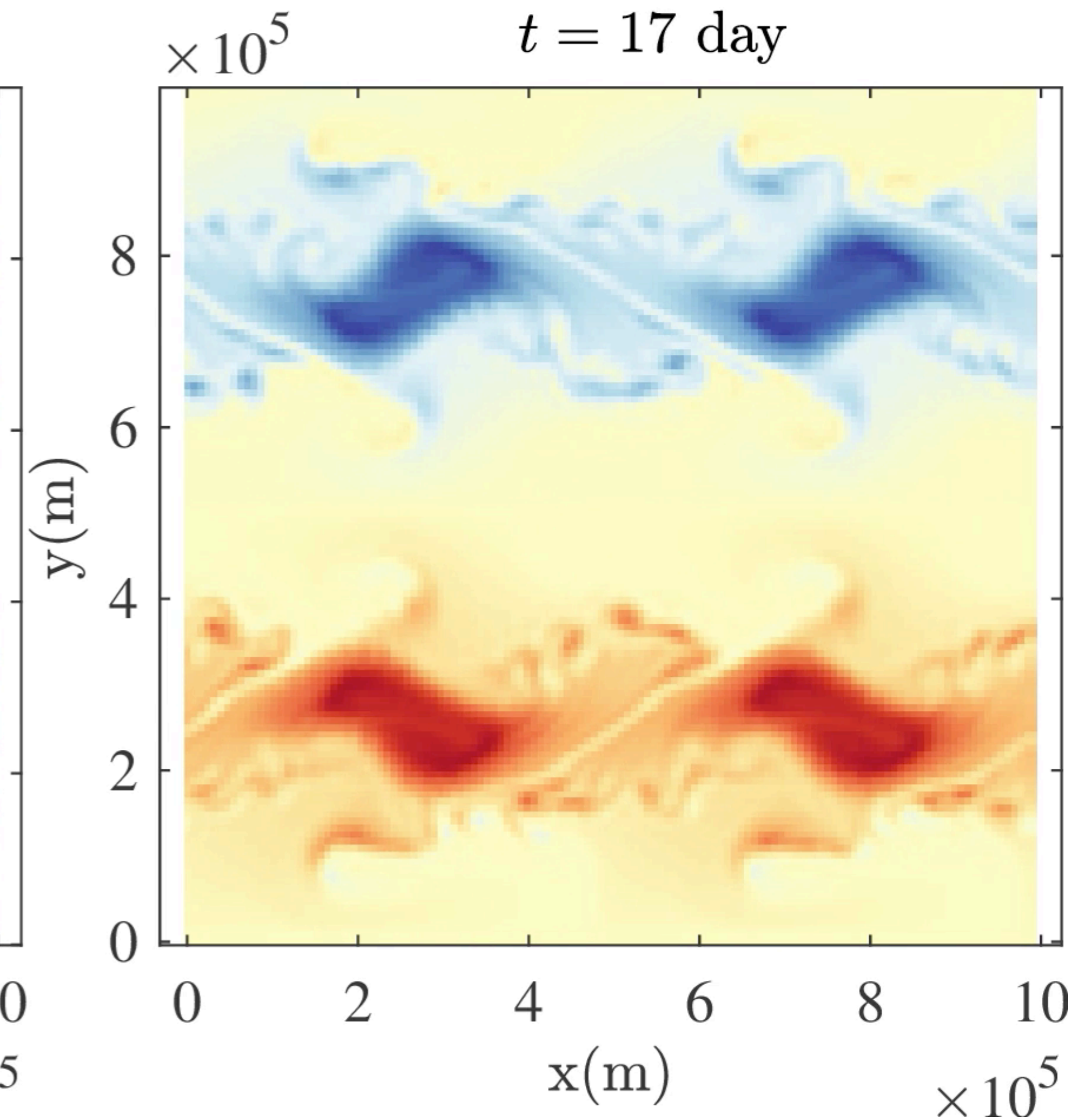
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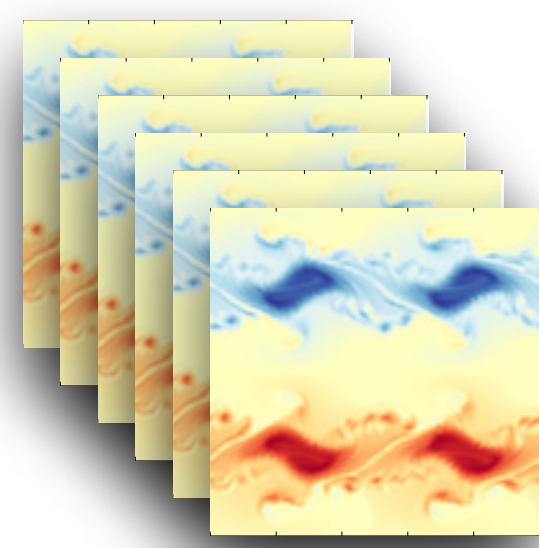


Deterministic 1024 x 1024

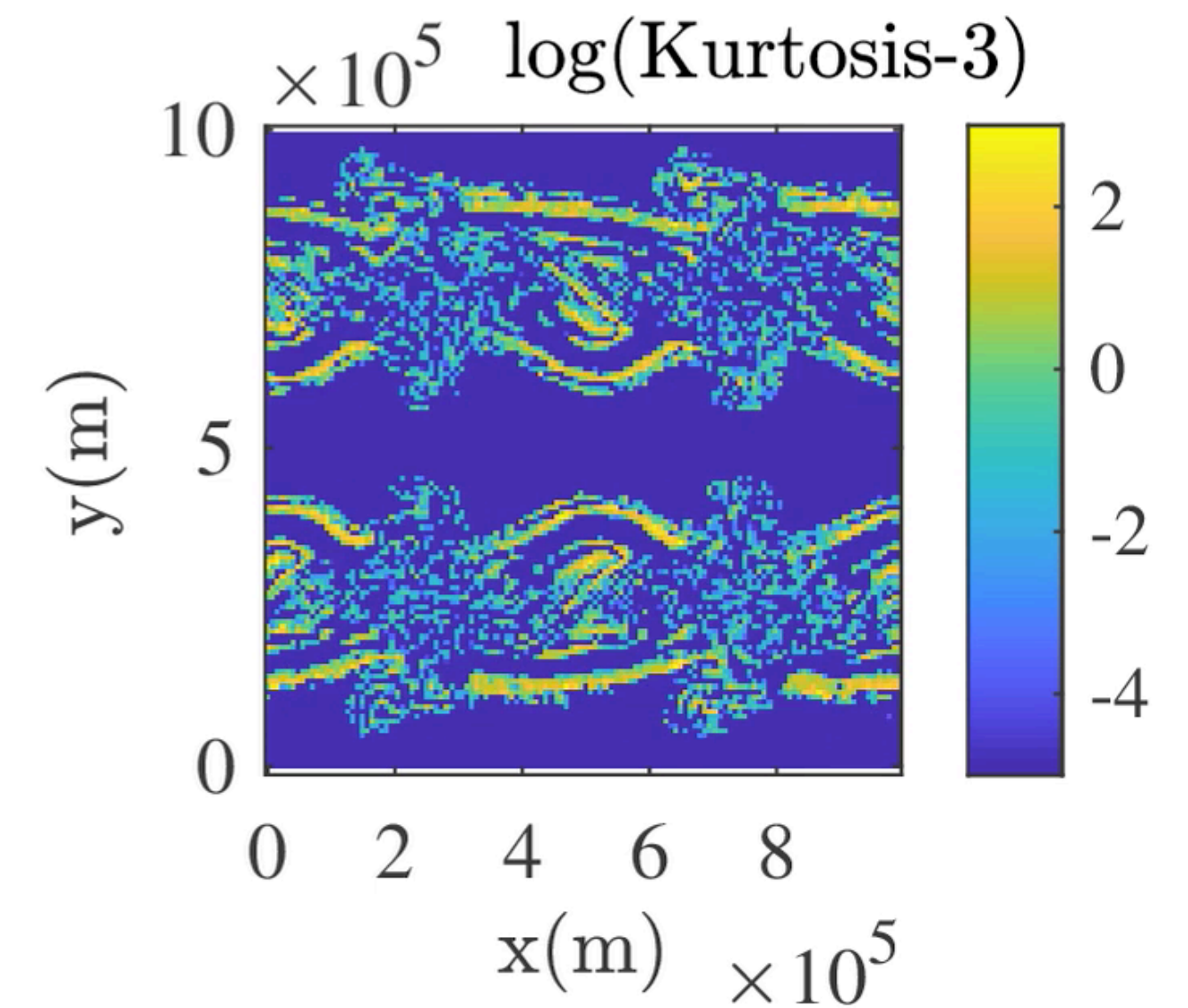
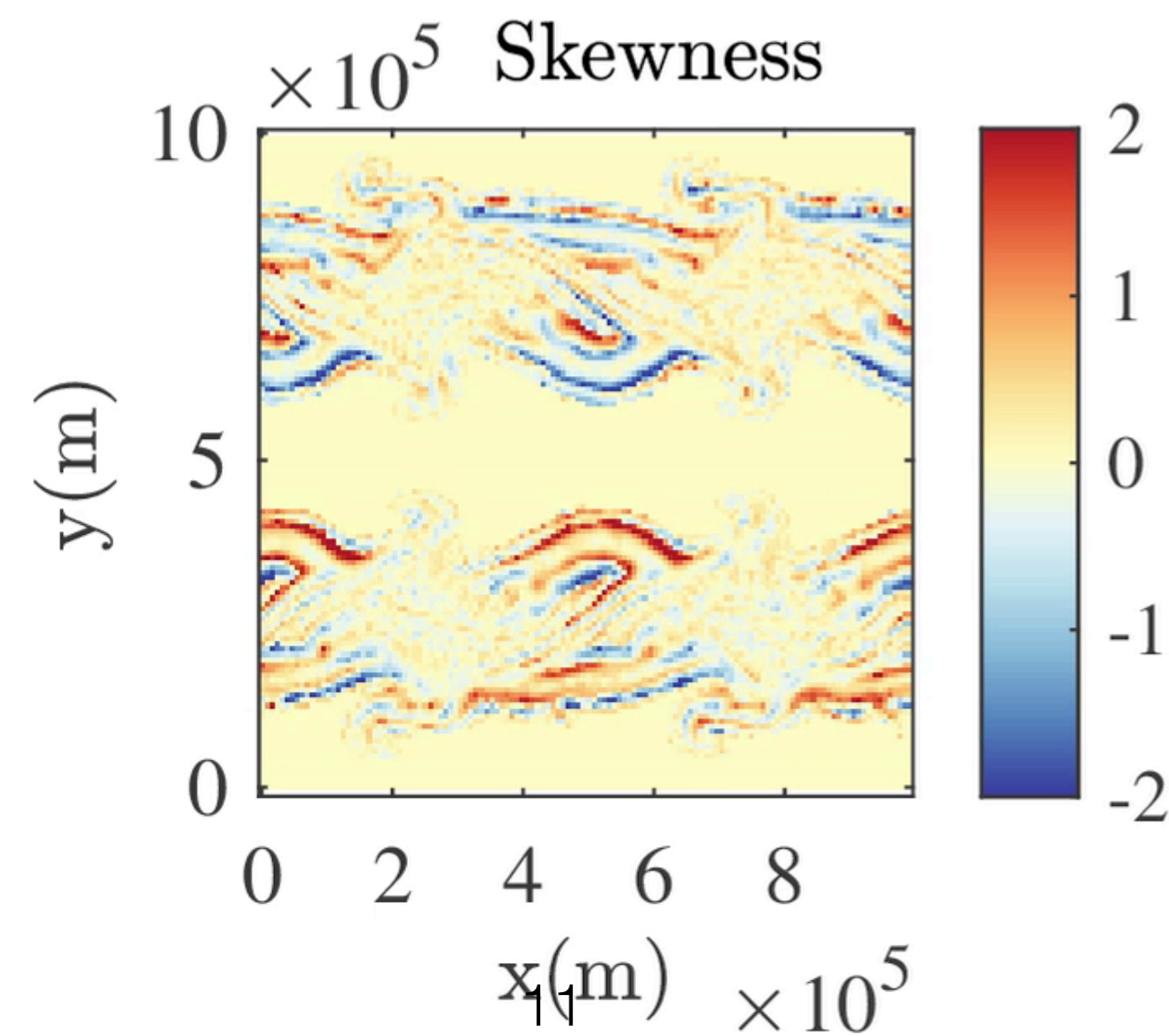
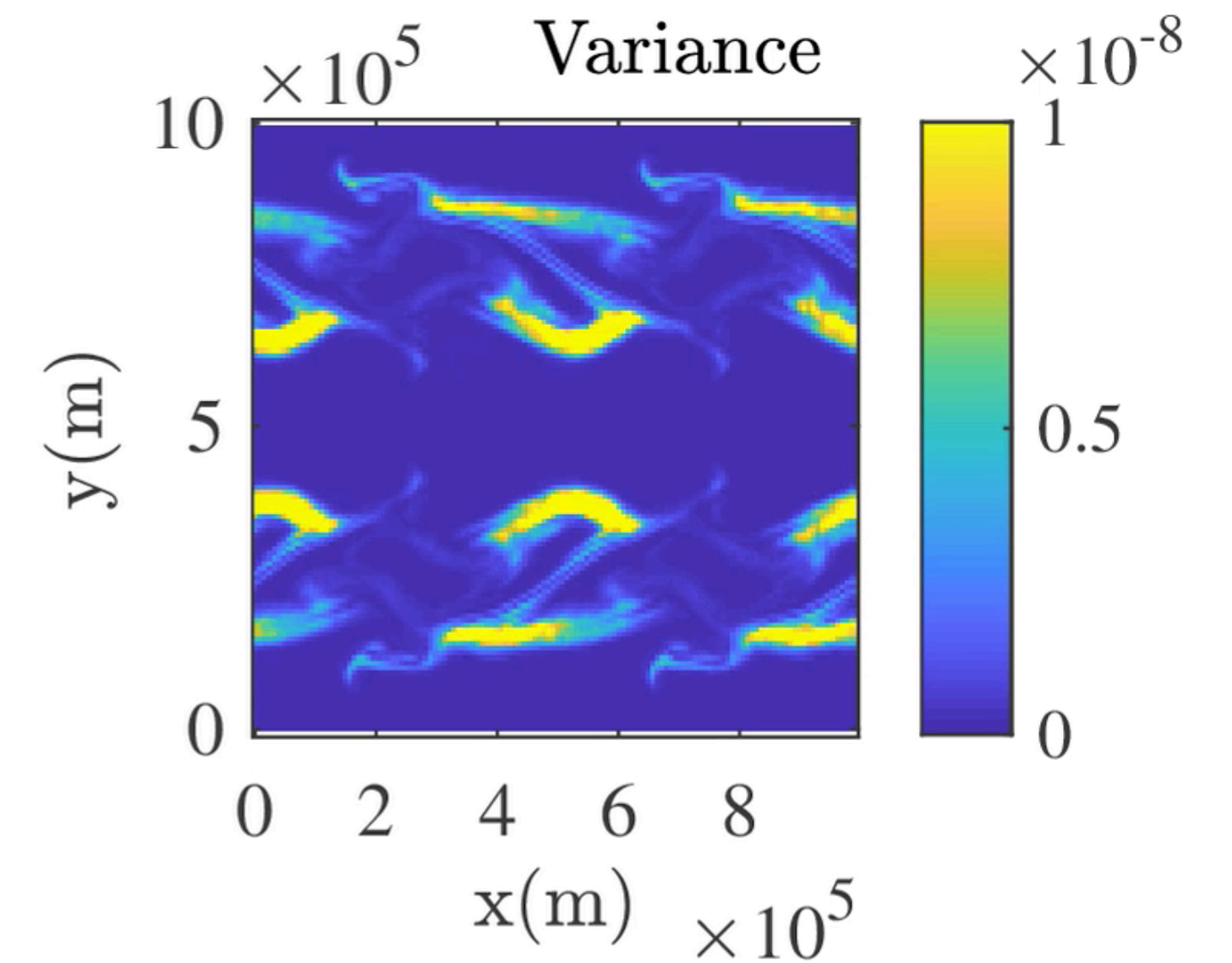
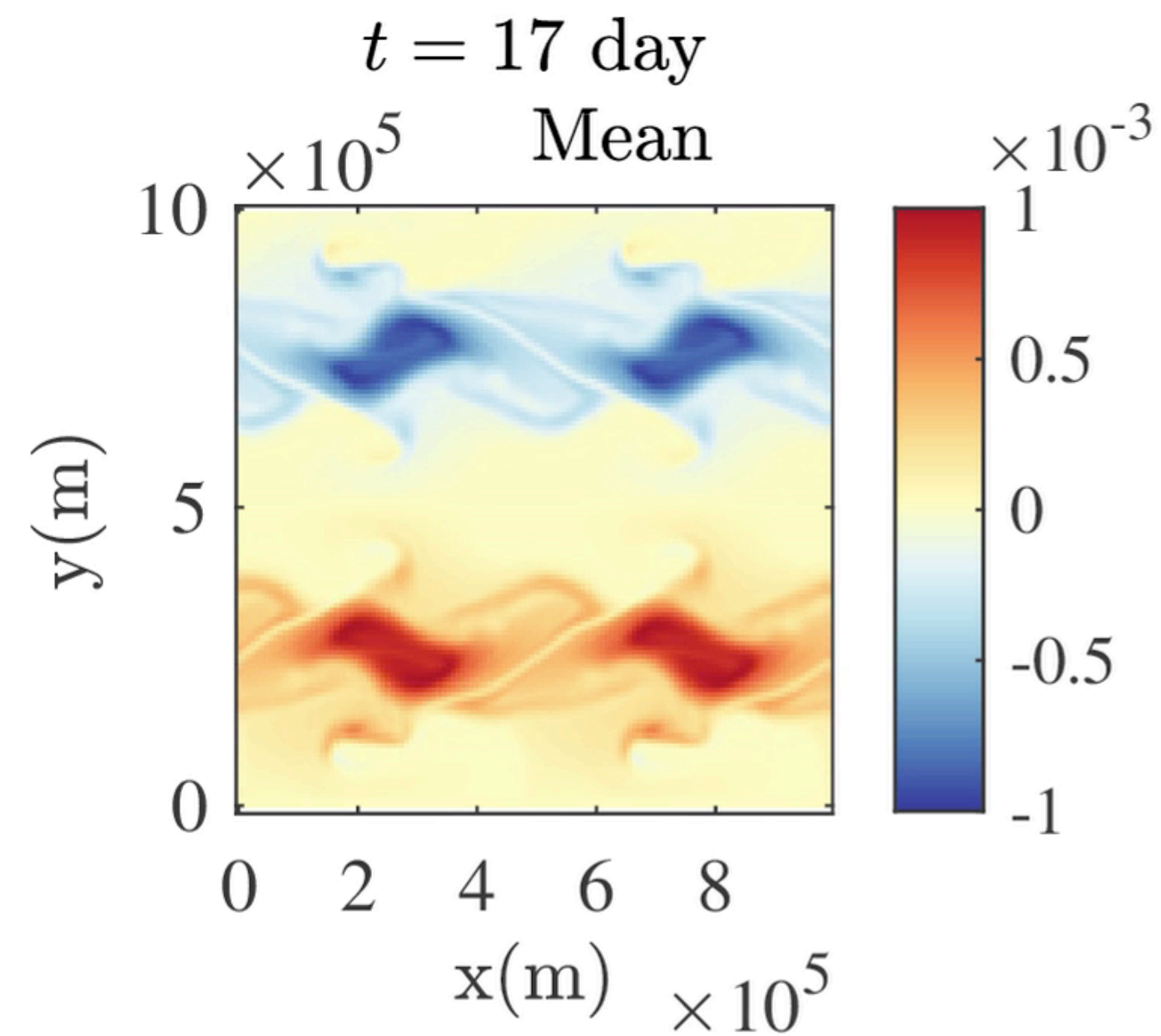


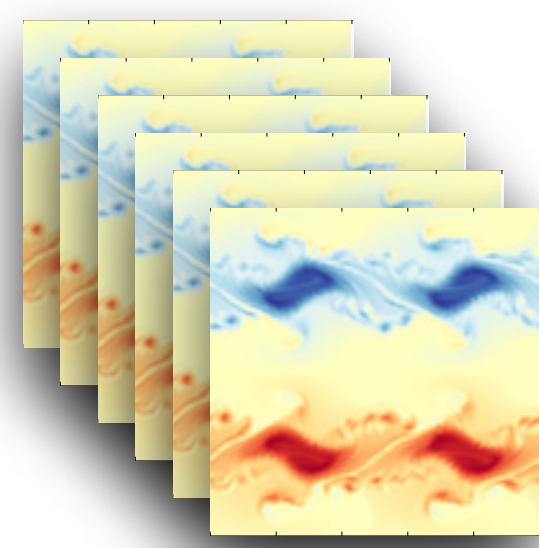
Location Uncertainty 128 x 128



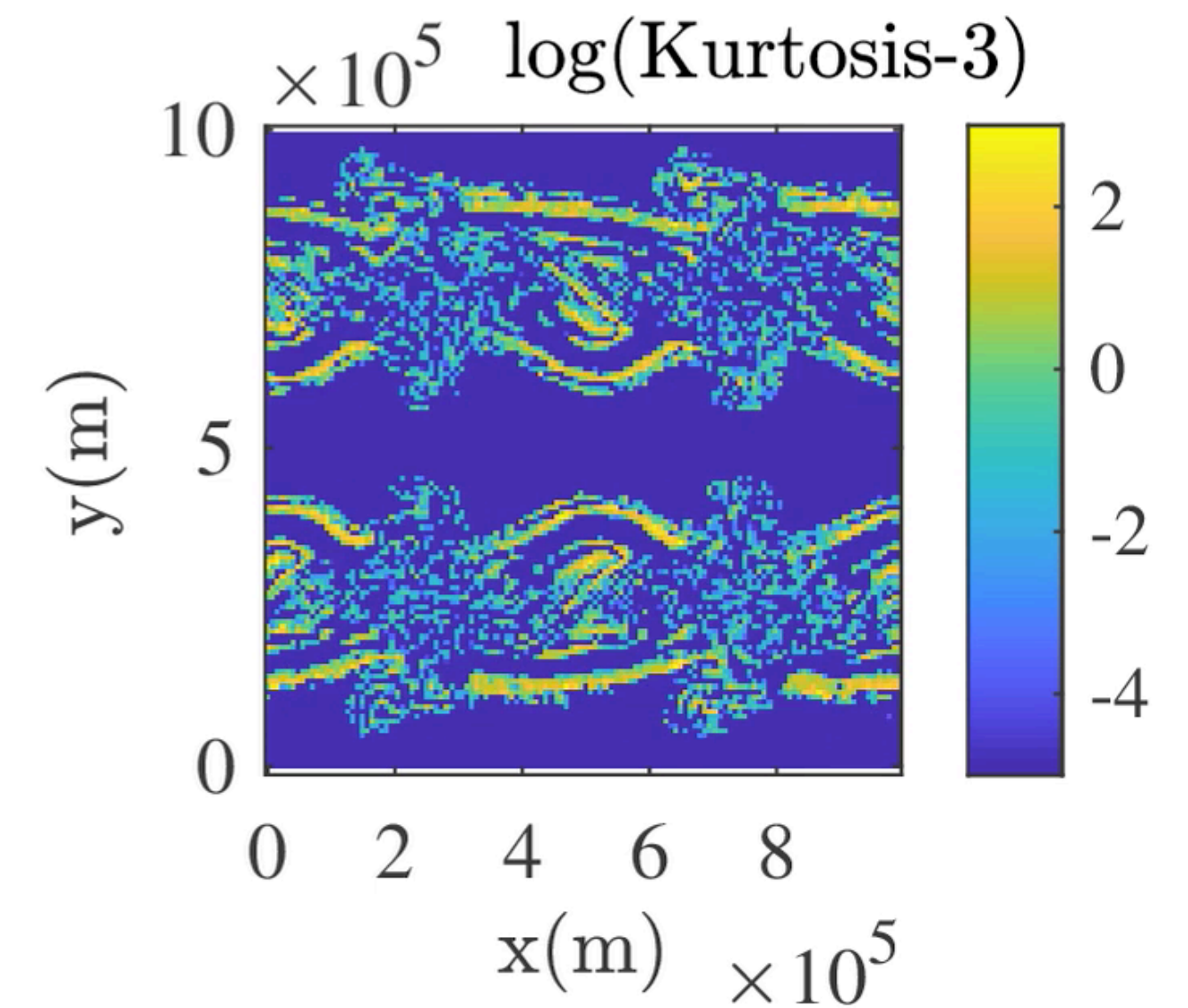
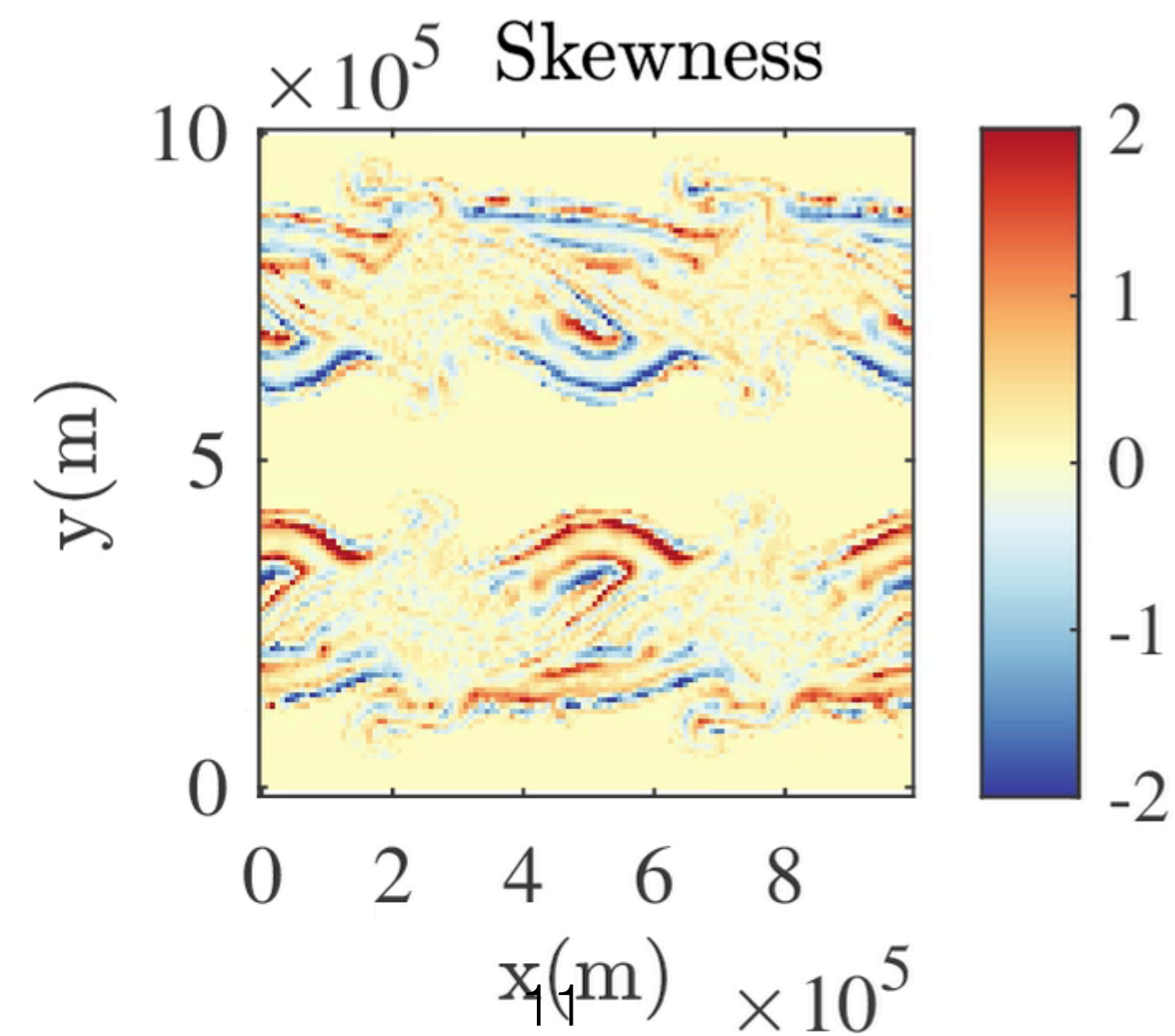
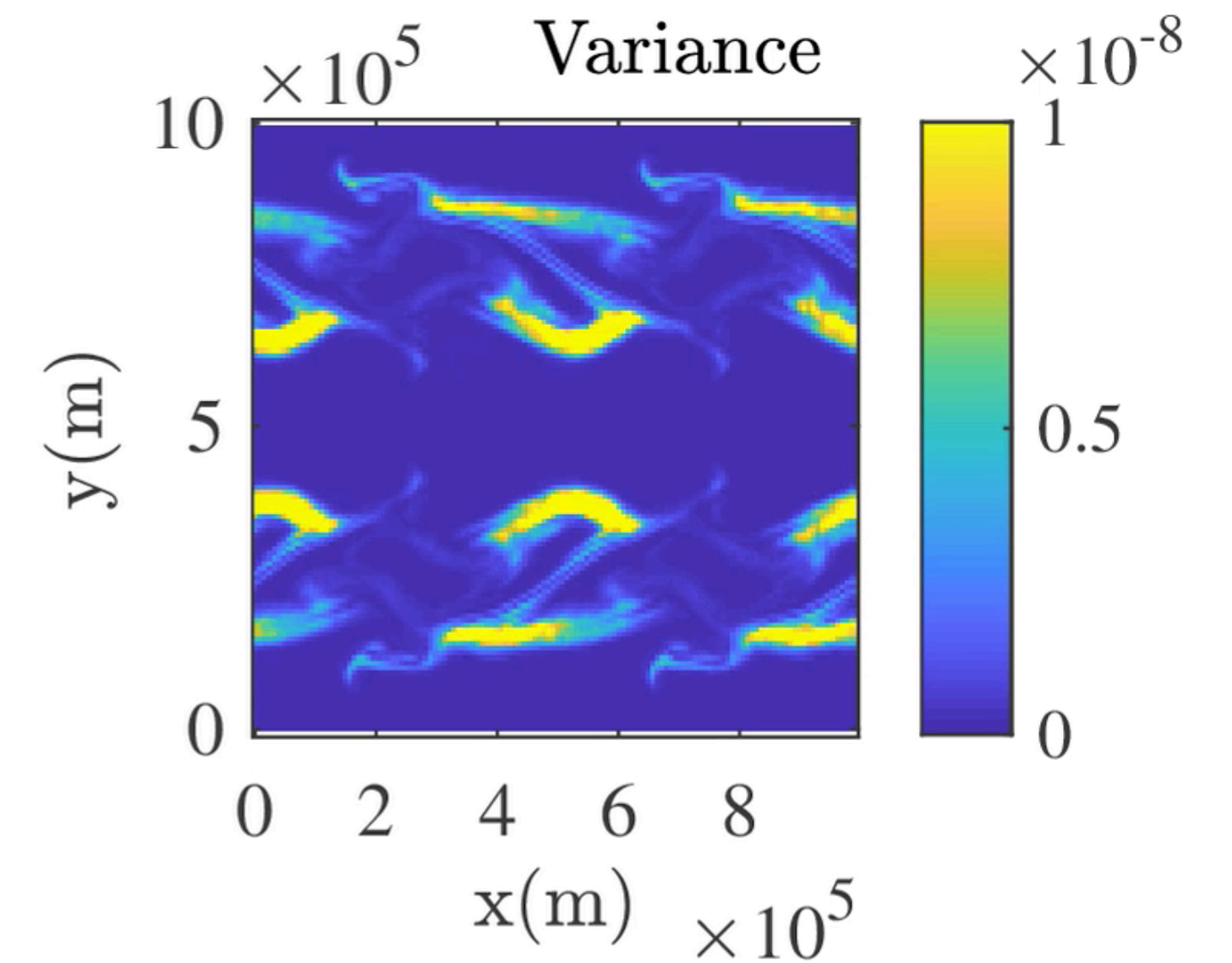
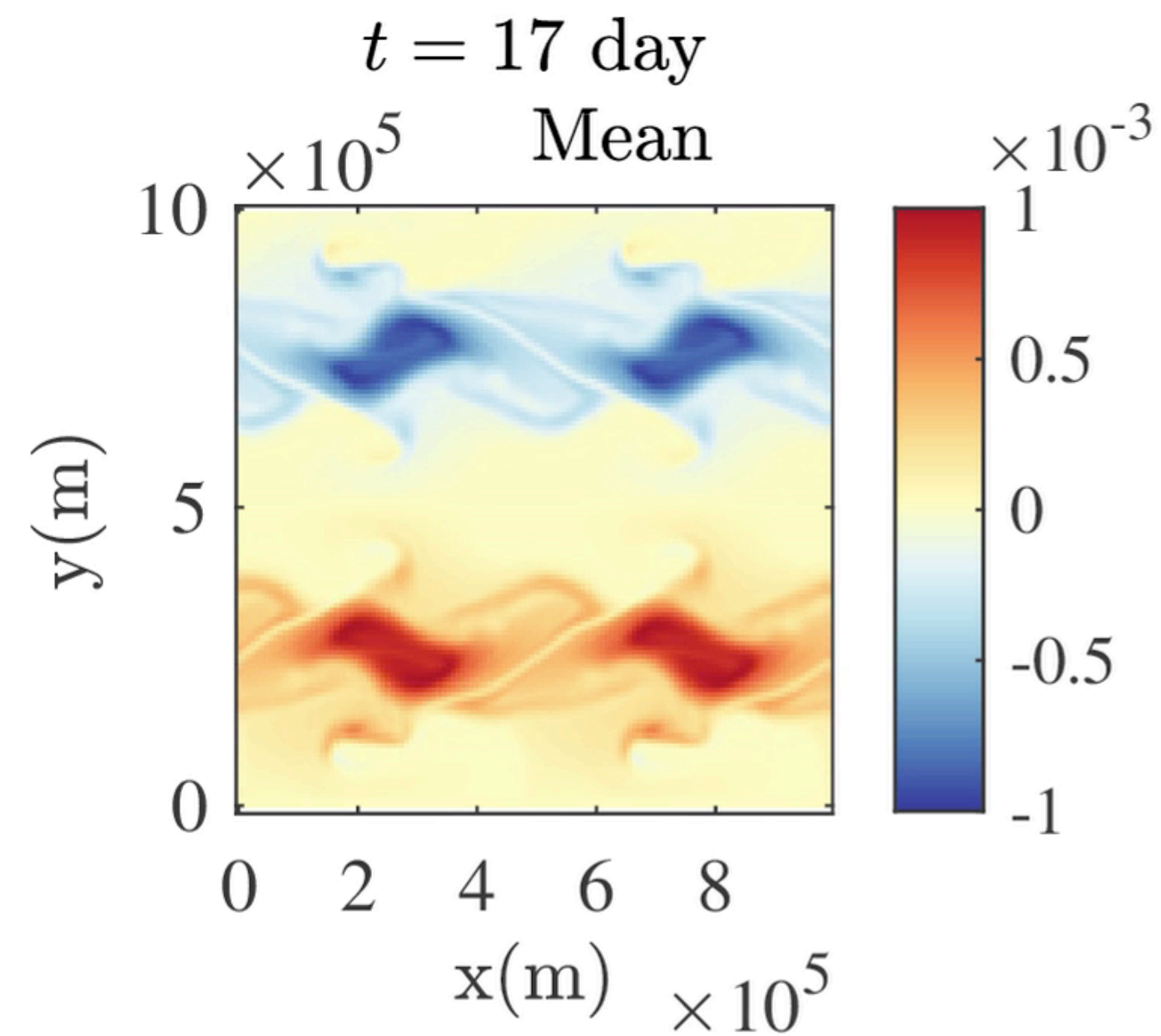


Ensemble: random coherent structures

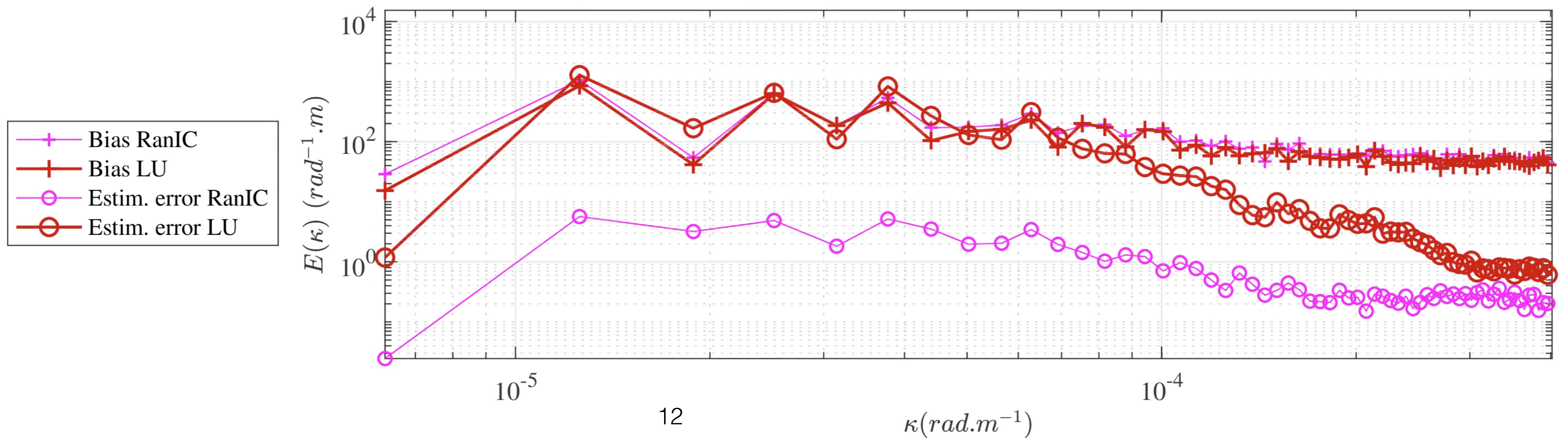
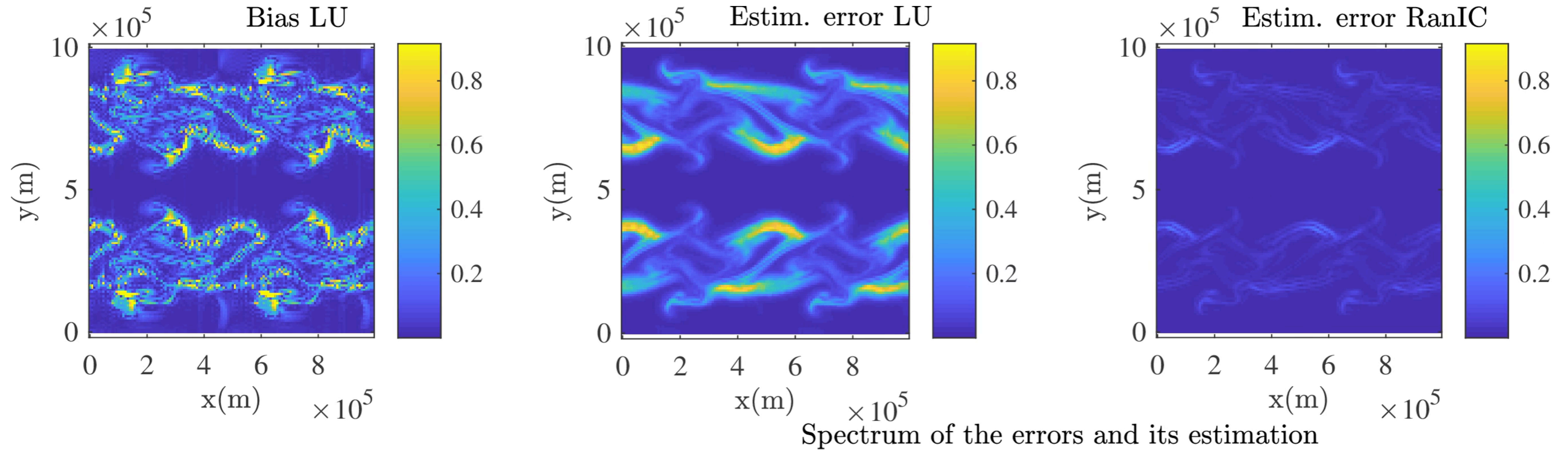




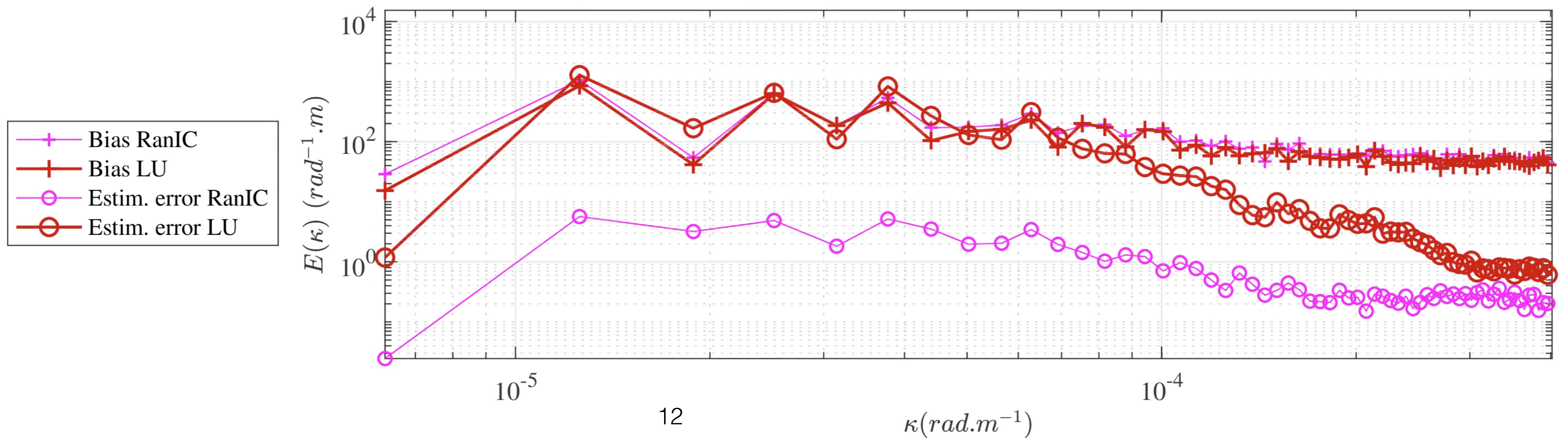
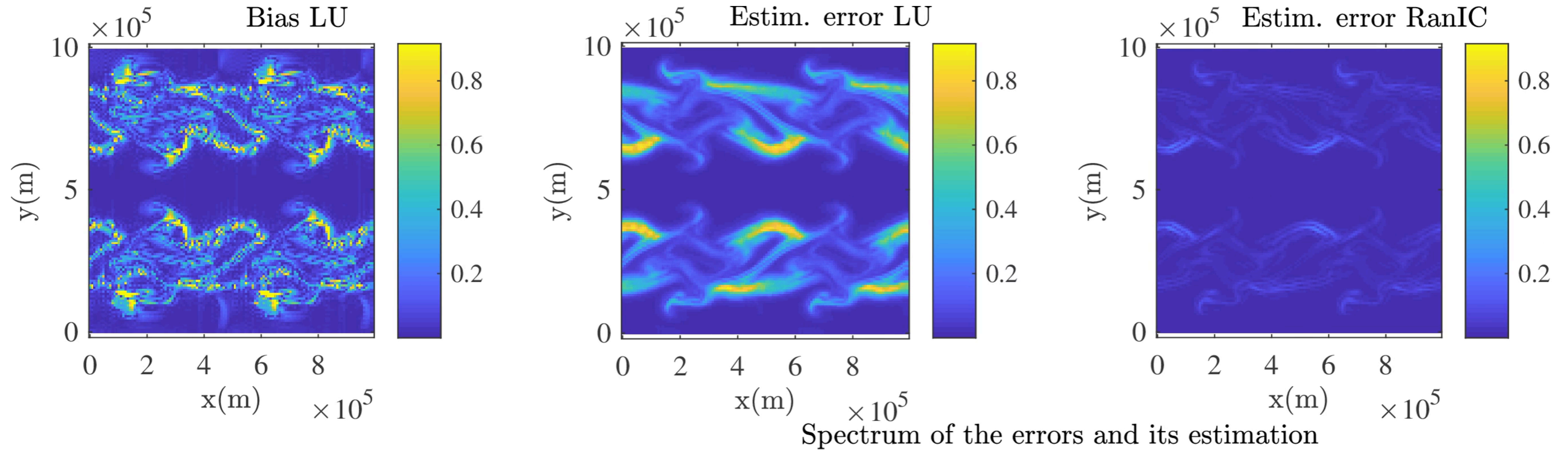
Ensemble: random coherent structures



Ensemble : uncertainty quantification



Ensemble : uncertainty quantification



Conclusion

Models under location uncertainty blindly describe unresolved triades

- Conserve energy
- Model derivation
- Instabilities triggered,
possibly followed by extreme events
- Uncertainty quantification to address filter divergence

Related works and outlooks

- Bifurcations (SQG) and attractor (Lorenz 63) exploration
- Stabilization / destabilization of Reduced Order Model
- Comparisons with data-driven σ and Stochastic Lie Derivative approaches (Holm and coauthors)
- Parametrization and tests based on higher-order statistics (curvature, energy flux through scales, bispectrum, ...)
- Mimic barotropization
- Girsanov theorem for MLE and Bayesian estimations with satellite images
- Learning σ on SWOT data
- Filtering / smoothing with (reduced) models under location uncertainty